DJ Journal of Engineering and Applied Mathematics, Vol. 4(2) 2018, pp. 1-12

RESEARCH ARTICLE

A Model for Imperfect Production System with Probabilistic Rate of Imperfect Production for Deteriorating Products

*Muhammad Waqas Iqbal[1], Biswajit Sarkar[1]
[1]Department of Industrial & Management Engineering, Hanyang University, Ansan,Gyeonggi-do, 15588, South Korea.

Received- 7 February 2018, Revised- 20 February 2018, Accepted- 24 February 2018, Published- 1 March 2018

ABSTRACT

Imperfection in a production process is inevitable. The rate of imperfect production does not remain constant but varies depending on several production parameters. Besides, some products, food products for example, have a tendency to deteriorate with time. The rates of deterioration of such products generally increase with time. For such an imperfect production system for deteriorating products, the decisions for the rate of production and planning horizon are very important. Keeping in view the importance, this study proposes an integrated production model of an imperfect production system for deteriorating products. The defective and the deteriorated items are separated and disposed through proper channel to fulfill environmental legislative requirements. The objective of the study is defined through a mathematical model for total cost per unit time, which is minimized by obtaining optimal values of the production rate and the cycle time. These variables are decided in such a way that the customer demand during the cycle is fulfilled by compensating the imperfect and deteriorated quantity. The proposed model is verified through some numerical examples.

Keywords: Imperfect production system, Deterioration, Probabilistic rate, Time-varying rate, Non-linear programming.

1. INTRODUCTION

Deterioration is a common phenomenon for food products. Production and inventory decisions vary largely due to decaying properties of these products. Common examples of such products include fresh fruits, vegetables, bakery items, and other food products [1-4]. Deterioration is caused by the physical/chemical decomposition of the products and the rate of decomposition usually increases with time. Furthermore, the production systems are not always perfect. There are several factors, which cause a production system to go into "out of control" state and cause imperfect production. In such a system, where production process is imperfect and products deteriorate with time, it becomes vital to precisely decide the production rate and the planning horizon. Considering such a system, this study focuses on deciding the optimal production rate and cycle time for deteriorating products being produced in an imperfect production system.

In literature, many researchers have investigated the effects of deterioration on inventory and production models. They have considered different conditions to develop mathematical expressions for the rate of deterioration. Earlier attempts of modeling an inventory system with deterioration include the research of [5]. Later, updated models according to real situation were formulated and by relaxing many other assumptions. [6] incorporated product shortages in deteriorating inventory system with time-dependent demand. He assumed a constant rate of deterioration. [7] added partial backlogging in an inventory system of deteriorating products with time-varying demand and a constant rate of deterioration. [8] further explored inventory systems and improved their previous model by adding a time-varying rate of deterioration with partial backlogging.

In real situations, products deteriorate with increasing rate. This idea was introduced by [9], where he proposed time-varying and maximum lifetime-dependent rate of deterioration. [10] described that products do not start deteriorating immediately after a specific period of time. They considered variable rate of deterioration. They also assumed that product demand is sensitive to selling-price and frequency of advertisement. [11] modeled an inventory system for deteriorating products and assumed that the rate of deterioration is linearly proportional to

*Corresponding author. Tel.: +821074981981
Email address: waqastextilion@gmail.com (M.W.Iqbal)
Double blind peer review under responsibility of DJ Publications
https://dx.doi.org/10.18831/djmaths.org/2018021001

time. [12] proposed an inventory system for deteriorating items and assumed time-varying rate of deterioration. They also considered the expiration time of the products at which the product is completely deteriorated.

[13] investigated an inventory system for deteriorating products where the rate of deterioration is time-varying and is inversely related to maximum lifetime of the product which is considered as product expiration time. In their model, they considered an offer of delay in payments to the customers. [14] discussed an inventory system considering deterioration of product quality and quantity simultaneously. They assumed that the rate of deterioration is linearly proportional to the time and exponentially proportional to the storage temperature. [15] developed an inventory model for deteriorating items and proposed that the rate of deterioration is exponentially related with time. They also assumed full and partial credit policy for retailer and customer respectively.

[16] proposed a production model with random rate of deterioration considering uniform, triangular and beta distributions. [17] considered retailer's inventory in a supply chain system for deteriorating products with time-varying and maximum lifetime-dependent rate of deterioration. [18] proposed a two-echelon supply chain system with random fuzzy rate of deterioration considering variable capital expenditure-dependent setup cost. [19] proposed an integrated production-inventory model for reverse logistics system considering time-varying rate of deterioration, production, demand, and product return. [2] developed closed loop supply chain model with benefit sharing considering decrease in value of outdated products as deterioration at a constant rate. [20] considered three stocks; manufacturing, remanufacturing, and returns in a reverse logistic system, with different quality levels of new and remanufactured products, deteriorating at time-varying rate. Moreover, the model on deteriorating products has been considered by [21-24].

During long run production, a common phenomenon is the production of defective items. The rate of production of defective items may be of two types, as constant defective rate and random defective rate. In constant defective rate, the total number of defective items is always fixed whereas in random defective rate, the number of defective items always varies based on several conditions of the production system. In reality, both the defective rates are varied based on the past data. It can be found constant defective rate [25] and random defective rate [26]. There is plenty of research since last four decades whereas the researchers are concentrating on the dependency of defective rate i.e. on which parts of the production system the defective rate depends. The pioneer attempt is as done in [27] and it proved a simple formula to calculate the number of defective items by using Maclaurin series expansion. [28] proposed an imperfect production model by considering a generalized function of the random defective rate, which depends on time of production, random time of movement from in control to out of control state and the production rate of the system.

[26] further studied an imperfect production system and modeled deteriorating production process assuming an optimal production run length. Working on the same grounds [29] allowed product shortages in their model. [30] considered randomness of time to machine failure in an imperfect production system. [31] extended the concept of imperfect production system to introduce a new research dimension by considering reduced selling price for defective/imperfect products. Reworking of the imperfect production items to make them as good as perfect quality items was introduced and modeled by [32]. [33] extended the model of Giri and Dohi [30] by adding safety stock and reliability parameter in an imperfect production system. [34] included time value of money in an EPQ system. [35, 36] studied deteriorating/imperfect production process which randomly shifts to "out of control state". Similarly, imperfection production models have been considered by [37-42].

Considering the above literature review, this study proposes an imperfect production model for deteriorating products. The rate of imperfect production is a random variable that follows normal, triangular, dual triangular, and beta distributions. The rate of deterioration increases with time and depends on maximum lifetime of the product. The total cost is minimized by obtaining optimal values of the production rate and cycle time.

2. PROBLEM DEFINITION, NOTATION AND ASSUMPTIONS

2.1. Problem definition

Figure 1 illustrates the process flow of the proposed integrated production system. Raw material is purchased and the product is produced at its production facility. The production system is imperfect and produces some defective items. The rate of defective production is considered as probabilistic, which follows different probability distributions. The produced items are stored as inventory in warehouse and are sold to the customers. Inventory items deteriorate at a specific rate, which increases with time. The defective and the deteriorated items are separated and forwarded to disposal center. The rate of production is taken as a decision variable, which fulfills the customer demand within a cycle by compensating the defective production as well as the deteriorated quantity.

This study suggests a mathematical model that estimates the optimal values of production rate and cycle time while minimizing the total cost.

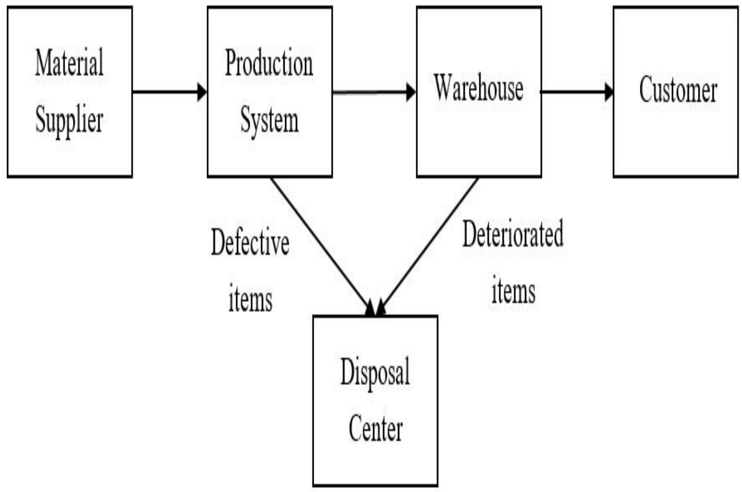

Figure 1.Process flow diagram

2.2. Notation

The abbreviations and notation for the mathematical model of proposed system is defined as follows:

2.2.1. Variables

The variables are given below:

T cycle time (time units)
p rate of production (units/unit time)

2.2.2. Parameters

The parameters were given below:

C_s setup cost per setup ($/setup)
C_{mt} material purchase cost per unit ($/unit)
C_p production cost per unit ($/unit)
h inventory holding cost per unit per unit time ($/unit/unit time)
C_d disposal cost per unit of defective and deteriorated items ($/unit)
TC total cost per unit time ($\$/unit$time)
d rate of demand per unit time (units/unit time)
D total demand per cycle (units/cycle)
θ rate of deterioration
η rate of defective production
L maximum lifetime of product (time units)
I_x on-hand inventory at any time t,$0 \le t \le t_1$ (units)
I_y on-hand inventory at any time t,$t_1 \le t \le T$ (units)
I total inventory carried per cycle (units/cycle)
P quantity produced per cycle (units/cycle)
N_{det} deteriorated items per cycle (units/cycle)
N_{def} defective items per production cycle (units/cycle)

2.3. Assumptions

Some of the assumptions are given below:
1. An imperfect integrated production model is considered for deteriorating items.
2. The rate of defective production is considered as probabilistic, which follows normal, triangular, double triangular, and beta distributions.
3. The items deteriorate at a rate, which increases with time and is a function of maximum lifetime of the product.
4. The rate of demand is constant and known over the planning horizon.
5. The defective and the deteriorated items are not repairable and are disposed.
6. Inspection cost is negligible.

3. MODEL DEVELOPMENT

The proposed model assumes that the rate of demand is d units per unit time. The manufacturer plans its production for a fixed period of time t_1 and sells the items for a variable span of time T . The rate of production is variable such that customer demand be fulfilled within planning horizon. The production system is imperfect and produces some defective items during production process. Keeping the rate of production as p , the rate of defective production is ηp. Therefore, the rate of production of perfect items is$(1-\eta)p$. The produced quantity of perfect items is saved as inventory, which fulfills the customer demand. The stored items deteriorate with time. The rate of deterioration is a function of product's lifetime and increases with time, as is provided below.

$$\theta = \frac{1}{1+L-t}$$

The produced quantity fulfills the customer demand by compensating the defective production. During the interval $0 \rightarrow t_1$, at any time t , p number of product items are produced, out of which $(1-\eta)$ are perfect and added to the inventory. From the stored inventory d number of items are demanded, and θI number of items are deteriorated and removed from the inventory. The level of inventory increases during the production time as the rate of production is higher than the accumulative rate of demand, deterioration and imperfect production. During interval $t_1 \rightarrow T$, there is no production, and inventory stock is depleted by the demand, deterioration and imperfect production. The change in the inventory level is illustrated in figure 2.

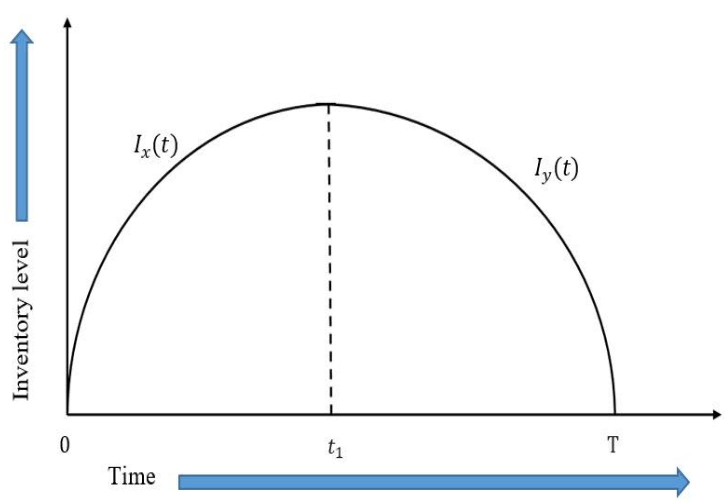

Figure 2.Behavior of inventory level

The governing differential equations of current inventory are expressed as in following equations.

$$\frac{dI_x(t)}{dt} = (1-\eta)p - d - \theta I_x, 0 \le t \le t_1 \quad (1)$$

$$\frac{dI_y(t)}{dt} = d - \theta I_y, t_1 \leq t \leq t_1 \tag{2}$$

The above differential equations are solved using the inventory conditions as given below and on-hand inventory is calculated at any time t. The level of inventory is zero when $t = 0$ or $t = T$. These inventory conditions are given as below:

$$I_x(t) = 0 \ \ at \ \ t = 0$$

,

$$I_y(t) = 0 \ \ at \ \ t = T$$

.

Considering the above inventory conditions, solution of (1) and (2) is as given below:

$$I_x(t) = ((1-\eta)p - d)(1+L-t)\left(ln\frac{1+L}{1+L-t}\right), 0 \leq t \leq t_1 \tag{3}$$

$$I_y(t) = d(1+L-t)\left(ln\frac{1+L-t}{1+L-T}\right), t_1 \leq t \leq t_1 \tag{4}$$

The system's total cost is incurred by the setup cost, material purchase cost, production cost, inventory holding cost and disposal cost.

Setup cost

The manufacturer, before starting production, prepares production equipment, moves the materials from material storage to the production floor, and prepares the warehouse for finished items. All such activities are named as production setup activities, which are managed for each production cycle. The cost incurred by these activities is termed as production setup cost, which is considered per manufacturer's cycle.

Setup cost per cycle = C_s

Cost of material

The quantity of material is arranged according to the produced quantity of the product. Therefore, requirement of material depends on the number of items produced in a cycle. The cost of material per cycle is calculated in below equation.

Material cost = C_{mt}P

where P is the number of items produced per cycle, as calculated below.

$$P = \int_0^{t_1} pdt = pt_1$$

Cost of production

The cost of machinery, labor, energy and overheads cumulatively are termed as production cost. The cost of production per cycle is calculated below.

Production cost = $C_p P$

Inventory holding cost

Customer demand is fulfilled from the stored inventory. The operations to keep the inventory of items in warehouse incur the inventory holding cost, which is calculated below.

Inventory holding cost = hI

where the value of I is calculated by using (3) and (4) and is provided below.

$$I=\int_0^{t_1} I_x(t)dt+\int_{t_1}^{T} I_y(t)dt$$
$$=\frac{1}{4}\Bigg\{((1-\eta)p-d)\left((1+L)^2-(1+L-t_1)^2\left(1+2ln\left(\frac{1+L}{1+L-t_1}\right)\right)\right)$$
$$+dt(T-2L-2)+2d(1+L)^2ln(1+L-t_1)-dt_1(2+2L-t_1)$$
$$\left(2ln\left(\frac{1+L-t_1}{1+L-T}\right)-1\right)-2d(1+L)^2ln(1+L-T)\Bigg\}$$

Disposal cost

The defective and deteriorated items during a cycle are disposed through a proper channel, such that the legislative environmental requirements be fulfilled. In order to calculate the disposal cost per cycle, number of defectives items and number of deteriorated items per cycle are calculated below.

Number of defective items per cycle

$$(N_{def})=\int_0^{t_1}\eta p dt=\eta p t_1$$

Similarly, number of items deteriorated per cycle is calculated below.

$$N_{det}=\int_0^{t_1}\theta I_x(t)dt+\int_{t_1}^{T}\theta I_y(t)dt$$
$$=((1-\eta)p-d)\left((1+L_1-t_1)ln\left(1+\frac{t_1}{1+L_1-t_1}\right)-(1+L_1-T)ln\left(1+\frac{T}{1+L_1-T}\right)+T-t_1\right)$$
$$+d(1+L_1)ln(1+l_1)-d(1+L_1)ln(1+L_1-t_1)+dt_1ln\left(1+\frac{T-t_1}{1+L_1-T}\right)-dt_1$$

Total cost of disposal per cycle is calculated in the following equation.

Disposal cost $=C_d(N_{def}+N_{det})$,

Total cost

The rate of defective production η is a random variable, expected value of which depends on the type of distribution it follows.

For this study, four different probability distributions are considered, expected values of which are described below.

Case 1: Normal distribution

$$E(\eta)=\frac{a+b}{2}$$

Case 2: Triangular distribution

$$E(\eta)=\frac{a+b+c}{3}$$

Case 3: Double triangular distribution

$$E(\eta)=\frac{a+4b+c}{6}$$

Case 4: Beta distribution

$$E(\eta)=\frac{\alpha}{\alpha+\beta}$$

The total cost per unit time of the given system is calculated in following equation:

$$TC(p.T)=\frac{1}{T}\{C_s+(C_{mt}+C_p)P+hI+C_d(N_{def}+N_{det})\}$$

$$\begin{aligned}TC(p.T)=\frac{1}{T}\Bigg\{&C_s+(C_{mt}+C_p)pt_1\\&+\frac{h}{4}\Bigg\{((1-\eta)p-d)\left((1+L)^2-(1+L-t_1)^2\left(1+2\ln\left(\frac{1+L}{1+L-t_1}\right)\right)\right)\\&+2d(1+L)^2ln(1+L-t_1)-dt_1(2+2L-t_1)\left(2ln\left(\frac{1+L-t_1}{1+L-T}\right)-1\right)\\&-2d(1+L)^2ln(1+L-T)+dT(T-2L-2)\Bigg\}\\&+C_d\Bigg\{((1-\eta)p-d)\Bigg((1+L_1-t_1)ln\left(1+\frac{t_1}{1+L_1-t_1}\right)\\&-(1+L_1-T)ln\left(1+\frac{T}{1+L_1-T}\right)+T-t_1\Bigg)d(1+L_1)ln(1+L_1)\\&-d(1+L_1)ln(1+L_1-t_1)+dt_1ln\left(1+\frac{T-t_1}{1+L_1-T}\right)-dt_1+\eta pt_1\Bigg\}\Bigg\}\end{aligned}$$

The value of total cost per unit time varies with respect to the probability distribution followed by the rate of defective production. The objective of this research is to find the optimal values of the suggested decision variables, such that the total cost per unit time is minimized. The objective is expressed below.

Minimize $TC(p,T)$

subject to the following constraints:

$$p-N_{def}-N_{det}\geq D \tag{5}$$

$$p,T\geq 0 \tag{6}$$

where,

$$D=\int_0^T (d)dt=dT$$

Objective function TC(p,T) is the total cost per unit time. Constraint (6) ensures that the customer demand during complete cycle is fulfilled. Constraint (7) shows non-negativity of the decision variables.

4. NUMERICAL EXPERIMENTS

Numerical experiments are carried out to validate the proposed model. Four different examples are solved by considering different probability distributions for the rate of defective production. The values of the related parameters are taken from [16]. Mathematica 9 is used to obtain the optimal solution. The parameters and the optimum results are summarized below.

Case 1: For this case, the rate of defective production is considered to be following the normal distribution. The values of input parameters are; a=0.15, b=0.25, C_s = \$800/setup, C_{mt} = \$8/unit, C_p = \$5/ unit, h=

\$0.6/ unit/month, $C_d = 1000$ units/month, L=10 months, $t_1 = 0.3$ month. The optimal solution is, $TC^* = \$14976$ /month, $T^* = 0.63$ month, $p^* = 2196$ units/month. The results are graphically illustrated in figure 3(a).

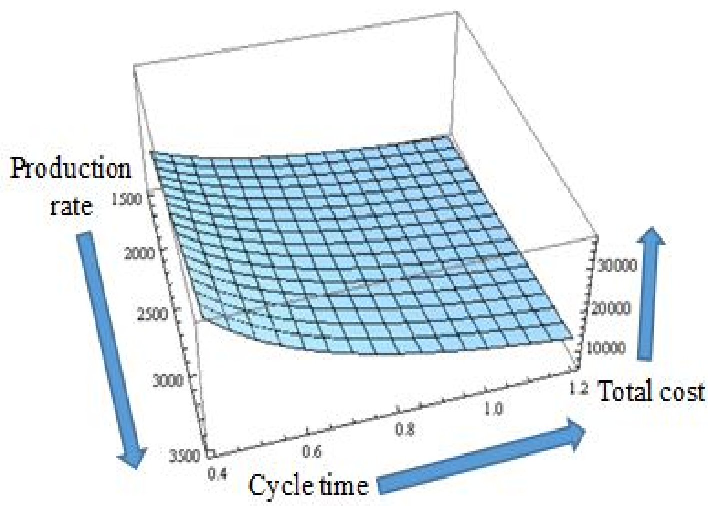

Figure 3(a).Graphical illustration of results for case 1

Case 2: For this case, rate of defective production is considered to be following the triangular distribution. The values of input parameters are; a=0.15, b=0.35, c=0.25, $C_s = \$800/$ setup, $C_{mt} = \$8/$ unit, $C_p = \$5/$ unit, h=\$0.6/ unit/month, $C_d = \$0.5/$ unit, d=1000 units/month, L=10 months, $t_1 = 0.3$ month. The optimal solution is, $TC^* = \$14960$/month, $T^* = 0.65$ month, $p^* = 2247$ units/month. The results are graphically illustrated in figure 3(b).

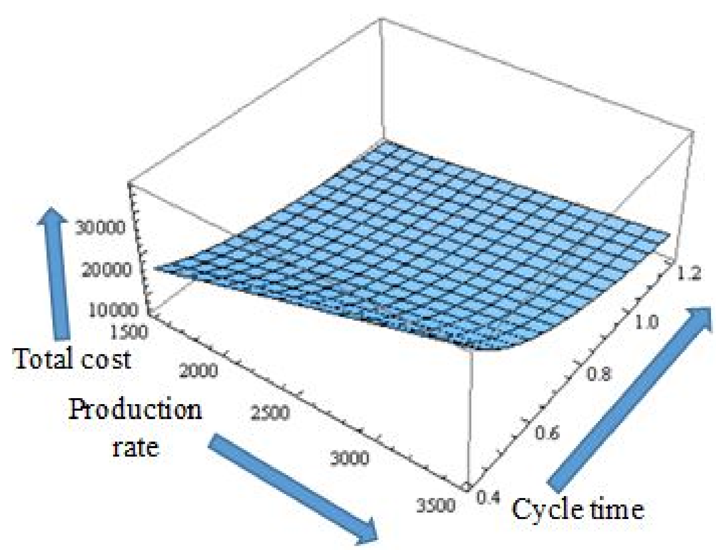

Figure 3(b).Graphical illustration of results for case 2

Case 3: For this case, rate of defective production is considered to be following the double triangular distribution. The values of input parameters are; a=0.15 , b=0.35,c=0.25, $C_s = \$800/$ setup, $C_{mt} = \$8$/unit, C_p =\$5/unit, h=\$0.6/unit/month, $C_d = \$0.5/$ unit, d=1000 units/month, L=10 months, $t_1 = 0.3$ month. The optimal solution is, $TC^* = \$14942$/month, $T^* = 0.67$ month, $p^* = 2303$ units/month. The results are graphically illustrated in figure 3(c).

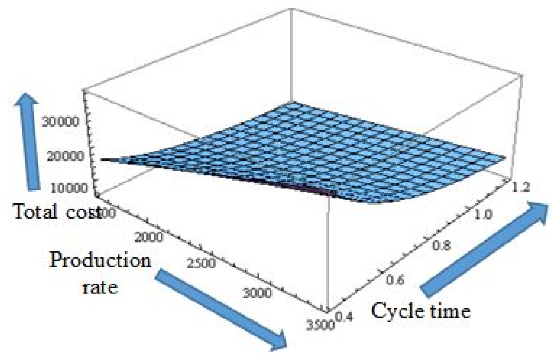

Figure 3(c).Graphical illustration of results for case 3

Case 4: For this case, rate of defective production is considered to be following the beta distribution. The values of input parameters are: $\alpha = 0.25, \beta = 0.35, C_s = \$800/$ setup, $C_{mt} = \$8/$ unit, $C_p = \$5/$ unit, h=\$0.6/ unit/month, $C_d = \$0.5/$ unit, d=1000 units/month, L=10 months, $t_1 = 0.3$ month. The optimal solution is, $TC^* = \$14889$/month, $T^* = 0.71$ month, $p^* = 2459$ units/month. The results are graphically illustrated in figure 3(d).

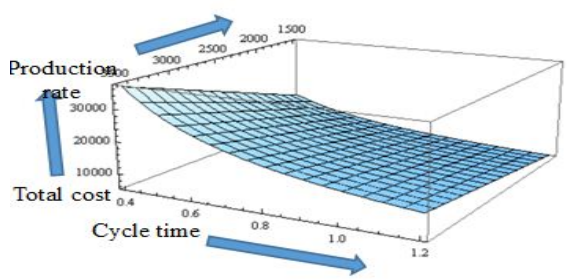

Figure 3(d).Graphical illustration of results for case 4

A cost comparison is provided graphically in figure 4 for different probability distributions for the random rate of defective production. We find that, for the considered cases, the total cost is minimum when rate of defective production follows beta distribution.

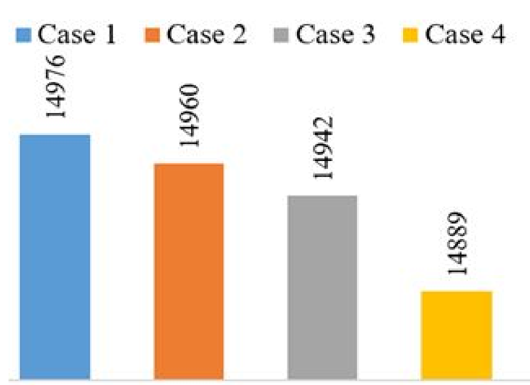

Figure 4.Cost comparison for different distributions

5. CONCLUSION

This study proposes an imperfect integrated production model. The rate of imperfect production is a random variable, which followed normal, triangular, dual triangular, and beta distribution. The products under consideration are deteriorating in nature. The rate of deterioration is a function of product's lifetime and increased with time. Four different models of total cost are provided depending on the type of probability distribution followed by the rate of imperfect production. The model is complemented with a constraint, which ensured that the customer demand is fulfilled by compensating the defective and deteriorated quantity. The optimal values of the production rate and the cycle time are computed, such that the total cost is minimum. The obtained results are compared, which reveals that the total cost is minimum when the random rate of imperfect production followed the beta distribution. The proposed model can be extended by adding shortages and backlogs. Moreover, the inspection cost and repairing process for imperfect items can be incorporated.

REFERENCES

[1] J.T.Teng, L.E.Cardenas-Barron, H.J.Chang and J.Wu, Y.Hu, Inventory Lot-Size Policies for Deteriorating Items with Expiration Dates and Advance Payments, Applied Mathematical Modelling, Vol. 40, No. 19-20, 2016, pp. 8605-8616,
https://dx.doi.org/10.1016/j.apm.2016.05.022.

[2] P.Yang, S.Chung, H.Wee, E.Zahara and C.Peng, Collaboration for a Closed-Loop Deteriorating Inventory Supply Chain with Multi-Retailer and Price-Sensitive Demand, International Journal of Production Economics, Vol. 143, No. 2, 2013, pp. 557-566,
https://dx.doi.org/10.1016/j.ijpe.2012.07.020.

[3] B.Mohammadi, A.A.Taleizadeh, R.Noorossana and H.Samimi, Optimizing Integrated Manufacturing and Products Inspection Policy for Deteriorating Manufacturing System with Imperfect Inspection, Journal of Manufacturing Systems, Vol. 37, 2015, pp. 299-315,
https://dx.doi.org/10.1016/j.jmsy.2014.08.002.

[4] S.Saha, I.Nielsen and I.Moon, Optimal Retailer Investments in Green Operations and Preservation Technology for Deteriorating Items, Journal of Cleaner Production, Vol. 140, 2017, pp. 1514-1527,
https://dx.doi.org/10.1016/j.jclepro.2016.09.229.

[5] P.Ghare and G.Schrader, A Model for Exponentially Decaying Inventory, Journal of industrial Engineering, Vol. 14, No. 5, 1963, pp. 238-243.

[6] R.Sachan, On (T, S i) Policy Inventory Model for Deteriorating Items with Time Proportional Demand, Journal of the operational research society, Vol. 35, No. 11 1984, pp.1013-1019,
https://dx.doi.org/10.1057/jors.1984.197.

[7] H.J.Chang and C.Y.Dye, An EOQ Model for Deteriorating Items with Time Varying Demand and Partial Backlogging, Journal of the Operational Research Society, Vol. 50, No. 11, 1999, pp. 1176-1182.

[8] K.Skouri, I.Konstantaras, S.Papachristos and I.Ganas, Inventory Models with Ramp Type Demand Rate, Partial Backlogging and Weibull Deterioration Rate, European Journal of Operational Research, Vol. 192, No. 1, 2009, pp. 79-92,
https://dx.doi.org/10.1016/j.ejor.2007.09.003.

[9] B.Sarkar, An EOQ Model with Delay in Payments and Time Varying Deterioration Rate, Mathematical and Computer Modelling, Vol. 55, No. 3, 2012, pp. 367-377.

[10] N.H.Shah, H.N.Soni and K.A.Patel, Optimizing Inventory and Marketing Policy for Non-Instantaneous Deteriorating Items with Generalized Type Deterioration and Holding Cost Rates, Omega, Vol. 41, No. 2, 2013, pp. 421-430.

[11] B.Sarkar and S.Sarkar, An Improved Inventory Model with Partial Backlogging, Time Varying Deterioration and Stock-dependent Demand, Economic Modelling, Vol. 30, 2013, pp. 924-932,
https://dx.doi.org/10.1016/j.econmod.2012.09.049.

[12] J.Wu, L.Y.Ouyang, L.E.Cardenas-Barron and S.K.Goyal, Optimal Credit Period and Lot Size for Deteriorating Items with Expiration Dates under Two-Level Trade Credit Financing, European Journal of Operational Research, Vol. 237, No. 3, 2014, pp. 898-908,
https://dx.doi.org/10.1016/j.ejor.2014.03.009.

[13] S.C.Chen and J.T.Teng, Retailer's Optimal Ordering Policy for Deteriorating Items with Maximum Lifetime under Supplier's Trade Credit Financing, Applied Mathematical Modelling, Vol. 38, No. 15, 2014, pp. 4049-4061.

[14] Y.Qin, J.Wang and C.Wei, Joint Pricing and Inventory Control for Fresh Produce and Foods with Quality and Physical Quantity Deteriorating Simultaneously, International Journal of Production Economics, Vol. 152, 2014, pp. 42-48.

[15] B.Sarkar and S.Saren, Partial Trade-Credit Policy of Retailer with Exponentially Deteriorating Items, International Journal of Applied and Computational Mathematics, Vol. 1, No. 3, 2015, pp. 343-368.

[16] B.Sarkar, A Production-Inventory Model with Probabilistic Deterioration in Two-Echelon Supply Chain Management, Applied Mathematical Modelling, Vol. 37, No. 5, 2013, pp. 3138-3151.

[17] W.C.Wang, J.T.Teng and K.R.Lou, Seller's Optimal Credit Period and Cycle Time in a Supply Chain for Deteriorating Items with Maximum Lifetime, European Journal of Operational Research, Vol. 232, No. 2, 2014, pp. 315-321,
https://dx.doi.org/10.1016/j.ejor.2013.06.027.

[18] S.Priyan and R.Uthayakumar, An Integrated Production-Distribution Inventory System for Deteriorating Products Involving Fuzzy Deterioration and Variable Setup Cost, Journal of Industrial and Production Engineering, Vol. 31, No. 8, 2014, pp. 491-503,
https://dx.doi.org/10.1080/21681015.2014.994800.

[19] V.K.Mishra, Production Inventory Model for Deteriorating Items with Shortages and Salvage Value Under Reverse Logistics, International Journal of Mathematical Modelling & Computations, Vol. 2, No. 2, 2012, pp. 99-110.

[20] A.Bouras and L.Tadj, Production Planning in a Three-Stock Reverse-Logistics System with Deteriorating Items under a Continuous Review Policy, Journal of Industrial and Management Optimization, Vol. 11, No. 4, 2015, pp. 1041-1058.

[21] B.K.Sett, S.Sarkar, B.Sarkar and W.Y.Yun, Optimal Replenishment Policy with Variable Deterioration for Fixed-Lifetime Products, Scientia Iranica. Transaction E, Industrial Engineering, Vol. 23, No. 5, 2016, pp. 2318.

[22] B.Sarkar, B.Mandal and S.Sarkar, Preservation of Deteriorating Seasonal Products with Stock-Dependent Consumption Rate and Shortages, Journal of Industrial & Management Optimization, Vol. 13, No. 1, 2017, pp. 187-206.

[23] B.Sarkar, Supply Chain Coordination with Variable Backorder, Inspections, and Discount Policy for Fixed Lifetime Products, Mathematical Problems in Engineering, Vol. 2016, 2016, pp. 1-14.

[24] W.Iqbal and B.Sarkar, Recycling of Lifetime Dependent Deteriorated Products Through Different Supply Chains, RAIRO-Operations Research, 2017,
https://dx.doi.org/10.1051/ro/2017051.

[25] S.Khanra and K.Chaudhuri, A Note on an Order-Level Inventory Model for a Deteriorating Item with Time-Dependent Quadratic Demand, Computers & Operations Research, Vol. 30, No. 12, 2003, pp. 1901-1916,
https://dx.doi.org/10.1016/S0305-0548(02)00113-2.

[26] C.H.Kim and Y.Hong, An Optimal Production Run Length in Deteriorating Production Processes, International Journal of Production Economics, Vol. 58, No. 2, 1999, pp. 183-189,
https://dx.doi.org/10.1016/S0925-5273(98)00119-4.

[27] M.J.Rosenblatt and H.L.Lee, Economic Production Cycles with Imperfect Production Processes, IIE Transactions, Vol. 18, No. 1, 1986, pp. 48-55.

[28] S.S.Sana, A Production–Inventory Model in an Imperfect Production Process, European Journal of Operational Research, Vol. 200, No. 2, 2010, pp. 451-464.

[29] K.J.Chung and K.L.Hou, An Optimal Production Run Time with Imperfect Production Processes and Allowable Shortages, Computers & Operations Research, Vol. 30, No. 4, 2003, pp. 483-490,
https://dx.doi.org/10.1016/S0305-0548(01)00091-0.

[30] B.Giri and T.Dohi, Exact Formulation of Stochastic EMQ Model for an Unreliable Production System, Journal of the Operational Research Society, Vol. 56, No. 5, 2005, pp. 563-575.

[31] S.S.Sana, S.K.Goyal and K.Chaudhuri, An Imperfect Production Process in a Volume Flexible Inventory Model, International Journal of Production Economics, Vol. 105, No. 2, 2007, pp. 548-559,
https://dx.doi.org/10.1016/j.ijpe.2006.05.005.

[32] Y.S.P.Chiu, K.K.Chen, F.T.Cheng and M.F Wu, Optimization of the Finite Production Rate Model with Scrap, Rework and Stochastic Machine Breakdown, Computers & mathematics with applications, Vol. 59, No. 2, 2010, pp. 919-932.

[33] B.Sarkar, S.S.Sana and K.Chaudhuri, Optimal reliability, Production Lot Size and Safety Stock in an Imperfect Production System, International Journal of Mathematics in Operational Research, Vol. 2, No. 4, 2010, pp. 467-490,
https://dx.doi.org/10.1504/IJMOR.2010.033441.

[34] B.Sarkar and I.Moon, An EPQ Model with Inflation in an Imperfect Production System, Applied Mathematics and Computation, Vol. 217, No. 13, 2011, pp. 6159-6167.

[35] M.Suresh and N.Nisaantha Kumar, Determining the Most Probable Orientation of a Part using Centroid Solid Angle Method, Journal of Advances in Mechanical Engineering and Science, Vol. 1, No. 3, 2015, pp. 21-27,
http://dx.doi.org/10.18831/james.in/2015031003.

[36] B.Sarkar and S.Saren, Product Inspection Policy for an Imperfect Production System with Inspection Errors and Warranty Cost, European Journal of Operational Research, Vol. 248, No. 1, 2016, pp. 263-271,
https://dx.doi.org/10.1016/j.ejor.2015.06.021.

[37] A.A.Taleizadeh, H.Samimi, B.Sarkar and B.Mohammadi, Stochastic Machine Breakdown and Discrete Delivery in an Imperfect Inventory-Production System, Journal of Industrial & Management Optimization, Vol. 13, No. 3, 2017, pp. 1511-1535.

[38] B.Sarkar, B.K.Sett and S.Sarkar, Optimal Production Run time and Inspection Errors in an Imperfect Production System with Warranty, Journal of Industrial & Management Optimization, Vol. 14, No. 1, 2018, pp. 267-282.

[39] B.K.Sett, S.Sarkar and B.Sarkar, Optimal Buffer Inventory and Inspection Errors in an Imperfect Production System with Preventive Maintenance, The International Journal of Advanced Manufacturing Technology, Vol. 90, No. 1-4, 2017, pp. 545-560.

[40] C.W.Kang, M.Ullah and B.Sarkar, Optimum Ordering Policy for an Imperfect Single-Stage Manufacturing System with Safety Stock and Planned Backorder, The International Journal of Advanced Manufacturing Technology, 2017, pp. 1-12,
https://dx.doi.org/10.1007/s00170-017-1065-8.

[41] N.Cheikhrouhou, B.Sarkar, B.Ganguly, A.I.Malik, R.Batista and Y.H.Lee, Optimization of Sample Size and Order Size in an Inventory Model with Quality Inspection and Return of Defective Items, Annals of Operations Research, 2017, pp. 1-23,
https://dx.doi.org/10.1007/s10479-017-2511-6.

[42] Dhananjaya Reddy, Cascade and System Reliability for Exponential Distributions, DJ Journal of Engineering and Applied Mathematics, Vol. 2, No. 2, 2016, pp. 1-8,
http://dx.doi.org/10.18831/djmaths.org/2016021001.