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RESEARCH ARTICLE

Impact of Reparation for Imperfect Quality Items having Shortages in the System under Multi-Trade-Credit-Period

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ABSTRACT

Recently researchers have been focusing on the reparation of imperfect items for their models. Due to long-lasting benefits of the global market, there are many circumstances when the supplier is situated far away from the purchaser. There is a possibility that the procured items by the buyer are not in a perfect condition due to poor production system of supplier or damages during transportation. These defective quality items are still repairable. If the purchaser immediately exchanges these defective items with the seller, then the total cost of these items will increase. The feasible solution in this scenario is that these defective items are sent to some local rework store for reworking. After checking the quality of received lot at buyer's warehouse, the defective items are sent to a local store in a single batch for reparation. After reparation, these items are transported to the buyer's warehouse. It is assumed that there exists shortages in the system, when rework items are moved into the buyer's inventory. Additionally, the supplier also allows a multi-delay-in-payment for the buyer. An algebraic system is employed to make the optimal solution in closed form for each case. The proposed model has an objective to maximize the aggregate profit by creating a synergic repair effect for the defective quality product under multi-trade-credit-period policy and shortages. The present work shows the applicability of the developed model with numerical experiment and describes the managerial insights from sensitivity analysis.

Keywords: Defective item, Shortages, Delay-in-payment, Managerial insights, Non-derivative approach.

1. INTRODUCTION

Production managers make a lot of effort to plan efficient production systems to produce 100 % perfect items under economized cost. On the other hand, there is a possibility that the production system may produce some imperfect items. These bad quality items cut down the profit of the buyer and creates a negative influence on the environment. In order to purchase sustainable items at an economical rate in the global market, the buyer finds out the traders in global bourses and later do business with the best one. As distance between supplier and buyer is miles away, it is difficult for a supplier to dispatch a lot with all good items. Thus, for brand reputation and to confirm good quality, it is needed for the buyer to screen out an entire lot immediately as it is received. After screening of items, there may be probability that some fraction of items may be identified as imperfect items with small damage. These imperfect items are still repairable and have some value. The immediate interchange of defective items may not be feasible as the manufacturer is situated miles away. In several cases, the total cost of exchanging these items with supplier is not economical. These minor damage in these items can be fixed at the local repair shop. To maintain the sustainability at economical cost, reparation of these damaged items at local repair shop in comparison to replacing with seller is a feasible approach.

To be competitive, organizations work with other stakeholders as a mutual part of the business. As a result, managers are grasping the importance of co-operation with their buyers as well as suppliers. Competent financial structure with efficient inventory control plays an important part in the turnover of the organizations. In current scenarios, trade-credit-policy is a challenge for businesses. It becomes a win-win tactic for both parties. It further adds an extra opportunity cost or an interest rate for supplier and buyer, thus delayed payment time is a thoughtful theme that academics must focus to develop their inventory models. In conventional inventory model assumptions,

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purchaser immediately gives a payment of the product to the supplier, as soon as the products are sent to the buyer. On the other hand, in real corporate trades, the suppliers often agree to give delay-in-payment policy for the buyer to relax down the payments. This allowable time period for settling the payments minimizes the on-hand stock and expands the sales of the supplier. Also, the buyer sells the received items without giving back payment to the supplier and gets the additional interest income from these sales. The trade-credit financing is well-thought application to improve sales and maximize the profits of organizations. Due to this, interest income and opportunity cost are incorporated into the developed model to analyze the total profit.

The domain of inventory models has attained a lot of importance from scholars and experts in the industry. A main unrealistic assumption in the inventory structure by researchers is that each item in a received batch has characteristics attributes of desired value. The assumption of immediate payment of items is also relaxed for the developed model. Due to these impractical facts, a model with the imperfect quality item, reparation of these items under partial backordering and multi-trade-credit-policy is considered. To get an effective solution of the defined problem, an algebraic approach is employed for profit maximization. This study has an objective to get maximum profit. It also contributes a managerial insight for the industry with a supply of products such as home appliances and automobile manufactures. In many situations, the desired items are procured in advance from some global supplier. As a consequence, if some percentage of such received products is identified as imperfect, the replacement of such items is not possible immediately due to huge distances between them. Instead of increasing transportation cost and to avoid a negative influence on the environment, these defective items are reworked in the local repair shop. Moreover, in such situations, these organizations allows shortages which have backordered.

In this work literature review is described in section 2. Section 3 describes the problem definition with notation and assumptions of the developed model. The formulation of a mathematical model is shown in section 4. Section 5 and 6 determine the numerical example with detailed results and sensitivity analysis based on the effect of parametric changes respectively. Section 7 highlights the managerial insights. Section 8 concludes the paper with future research suggestions.

2. LITERATURE REVIEW

The theme of imperfect items has grown the attention of researchers in previous years. It is notable to state that, this is an impractical assumption that all items produced during manufacturing or in the received batch is based on the required quality. This assumption is unrealistic to satisfy and due to this, a restriction is raised in EOQ inventory model. [1] was the pioneer who discussed the connection between the percentage of imperfect items with produced lot when assembly system is out of control. [2] studied that batch size should be in small size to escape poor quality items. This is due to the fact that fraction of defects directly link with lot size. [3] considered the random percentage of imperfect items in a lot. Shortages are not allowed in their model and bad quality items are scrutinized by a screening procedure. [4] established a production model which considers an imperfect quality of product and lost in sales due to partial backordering. [5] proposed inventory model based on lot sizing in the production unit that considers rework and product defects. [6] proposed an EOQ model when fraction of given batch is not of desired quality. The stock-out situation takes place during screening of entire lot. Due to this stock-out condition, a percentage of demand is achieved by partial backordering and the leftover demand is meant to be lost sales. [7] studied the EOQ model and recommended that due to failure in inspection process, type I and type II error may exist. Also, these poor quality items and screening faults lead to shortages.

[8] introduced a single echelon in inventory system in the traditional EOQ framework with backordering to evaluate the quality impacts on cost factors performance. They revealed that earlier studies on imperfect quality items focus on deviations of the proportional yields. [9] presented a multiple-stage supply chain complexity under optimal control with defective products. As a result, it is deduced that integration of dumping decision and 100 % inspection of items is favorable and contributes a better coordination on reduced cost. [10] presented an extension of EOQ model with defective products and screening inaccuracies, as well as trade-credit-period policy is also considered. [11] structured an imperfect EPQ model with bi-level trade-credit-period having backordered demand. [12] developed an EPQ model by considering inspection errors and in result have to bear warranty cost for non-inspected products. In the case of total inspection, projected total cost with the addition of this screening cost resulted in higher inventory cost. [13] introduced an inventory model based on work-in-process by incorporating the random defective rate impact on lot size. [14] proposed a sustainable supply chain model where distance between nodes was the key consideration between buyer-supplier selections.

Trade-credit-policy is the understanding of financial approach provided by the supplier to the buyer. In this, the buyer is permissible with a time period to give back the payment for procured units without any interest. The buyer need not pay interest on warehouse units. In contrast, fee has to be paid if the payable amount for the procured items is not paid back under this given time. To fascinate the buyer in ordering the products in larger bulk batches, supplier normally offers this strategy. The existence of trade-credit-policy increases supplier's sales and lowers the stock level. [15] was the pioneer who presented permissible delay-in-payments with constant demand in inventory models. [16] studied an EOQ model with multi-level delay-in-payments. [17] analyzed an inventory model by considering trade-credit-policy. [18] proposed an integrated inventory framework for supplier-buyer that has flexible production system and imperfect items with partial trade-credit-policy. [19] examined the influence of maintenance policy on inadequate production system with trade-credit-policy. Supply chain model for multi-echelon with trade-credit-policy is established by taken into consideration of controllable lead time and backorder [20, 21]. [22] explored an inventory model with permissible shortages and trade-credit-policy for different conditions. [23] examined a supply chain network design with trade-credit-policy and partial backordering. [24] considered the effect of carbon emission and multi-delay-in-payments in global sustainable supply chain for multi-echelon. The previous works show that the trade-credit-policy among two parties is a joint bartering. In range of global trade, trade-credit-policy for inventory models has shown the consideration of many researchers.

From the mentioned literature review, this research considers the effects of reparation products for a defective lot by considering multi-delay-in-payments with partial backordering in inventory model for continuing the functional practice [25, 26]. The repaired items are received by the buyer when shortages are still available in the system. Also, in current business scenarios, trade-credit-policy is the foundation of short-term financing for organizations.

3. PROBLEM DEFINITION, NOTATION AND ASSUMPTIONS

This section elaborates problem statement with notations comprising decision variables, parameters and assumptions considered in the model.

3.1. Problem definition

The circumstances are established in which buyer procure the items from a merchant that is located globally. Mishandling of items during transportation or the expected lot with some imperfect items has to be checked. After screening of items, it is found that some percentage of items does not meet the requirements (desired quality). These defective items must be replaced by required quality items in order to satisfy the demand.

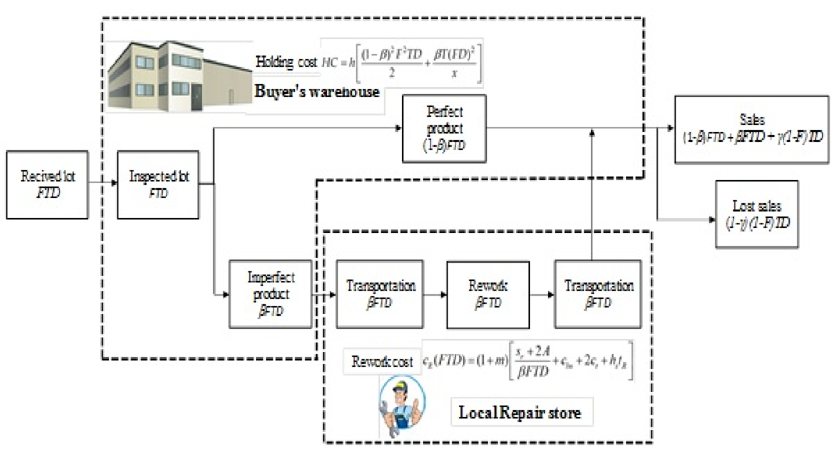

Figure 1.Inventory flow diagram of the imperfect quality product and rework inventory system with shortages

It is supposed that these imperfect items can be repaired at a local store. As the supplier is situated miles away, transportation cost of these exchanged items are high as compared to rework cost. It is likely to repair these defective items in a local repair store. After the completion of screening procedure, these imperfect items are sent to repair store instantly. According to the time framework, it is supposed that rework items arrived to the buyer's warehouse when shortages exist in the system. The local store includes fixed and variable costs, where the fixed cost comprises setup cost of the repair store and fixed transportation cost of the defective items. The variable cost covers labor, unit shipment and material costs per imperfect product and holding cost per repaired item at a local repair store. It is also assumed that the buyer is returning back the payment to provider with multi-trade-credit-period policy. As per the policy criteria with multi-delay-in-payments, the supplier allows the buyer a multi-payment allowable time. An interest is charged or earned within these multi-permissible periods. The given time to return payment influence the interest charged and earned. If the buyer sells the items before this permissible period, then an interest is earned from sales returns. In contrast, if buyer fails to give back the payment in the specified time, then an interest is charged according to the given interest rates. Hence, the objective of the developed model is to maximize the total profit by optimizing lot size (Q), optimizing the cycle time (T) and the fraction of cycle time (F). Figure 1 draws the inventory system of defective quality item and rework.

3.2. Notation

Decision variables

T cycle time (time unit)
F fraction of time in positive inventory level (%)

Dependent variable

Q Order size per cycle (units)

Parameters

D demand rate per unit time (units/time unit)
X inspection rate (units/time unit)
t_s inspection time of items (time unit)
t_R transportation, reparation and return back time of defective items (time unit)
t_T total transportation time by defective items (time unit)
R reparation rate (units/time unit)
A transportation fixed cost ($/trip)
B proportion of defective items (%)
O buyer ordering cost ($/order)
s_r setup cost of local repair store ($/setup)
h holding cost for imperfect items ($/unit/time unit)
h_r holding cost for reparation products ($/unit/time unit)
h_s holding cost at local repair store ($/unit/time unit)
C_s inspecting cost per unit ($/unit)
C_u buying cost for one unit ($/unit)
c_t transportation cost for the imperfect quality item per unit ($/unit)
c_{lm} material and labor cost needed to rework a unit product ($/unit)
l cost suffered due to sale loss ($/unit/time unit)
v penalty cost suffered due to goodwill loss ($/unit)
w proportion of imperfect quality items delivered to customers (%)
g unit return cost of the imperfect item ($/unit)
π backordered cost ($/unit/time unit)
P selling price for one unit ($/unit)
ρ percentage of backordered demand (%)
m markup percentage by local rework store (%)
X first allowable delay period (time unit)
Y second allowable delay period (time unit)
I_e interest earned (%)
I_{c1} interest charged (%)
I_{c2} interest charged (%)

The mathematical model is structured using the following assumptions.

3.3. Assumptions

1. The system includes one type of product.
2. The received lot may contain a fraction of imperfect items.
3. Shortages are permissible and partially backordered.

4. The demand for items and inspection rates are meant to be fixed and known.
5. The demand and screening process occur simultaneously, but the screening rate is faster than the demand rate (x > D).
6. Imperfect products can be repairable as they have minor damage in control system.
7. The fraction of imperfect items is known and given.
8. The holding cost of repaired products is more than the original holding cost of perfect products (h_R > h).
9. The repaired items are received when the shortage in the system still remains.
10. As the repaired lot is received, it first fulfills the backordered demand.
11. The provider permits a multi-trade-credit-period X and Y to the buyer. During these allowable periods, the buyer sells the item and uses its income to earn interest with a rate of I_e.
12. If the buyer fails to pay back to the supplier during first credit period X, then interest I_{c1} is applied, similarly, if the buyer fails to pay back during second allowable time Y to the supplier then extra interest is also applied at a rate of I_{c2}.
13. The proportion of defective products are sent to the customers which come back to the buyer in a fixed cycle. The buyer pays a unit cost for these reverted items and unit cost as penalty cost suffered due to goodwill loss.

4. MATHEMATICAL MODELLING

This segment describes an inventory model of total profit under multi-delay-in-payments, partial backordering, and reparation of defective items. The entire lot is examined at a rate x during screening time $t_s = I_{max}/x$. After screening t_s, the items which does not satisfy the desired quality are sent to the repair store. The repaired

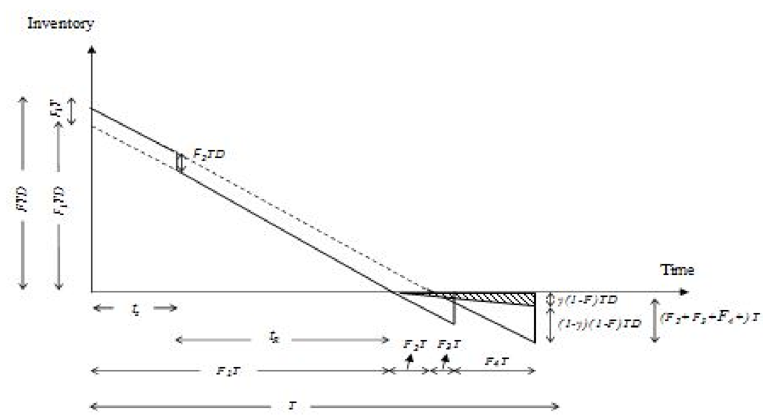

Figure 2.Inventory level with imperfect items and the return of rework items when shortages still exist

items are received back after t_R amount of time.

t_R is an aggregate time of transportation time and repair time. The repairing mechanism at the local repair store has control system. $s_r + 2A$ refers to the fixed cost that exist at repair shop, where s is taken as setup cost of the local repair store and A is considered as the fixed transportation cost. The variable cost is taken as $c_{lm} + 2c_t + h_s t_R$ per defective item at store, where c_{lm} is given as labor cost and material cost, h_s is holding cost at repair store, c_t is transportation cost, and t_R is total repair spell which comprises transportation time, return time, and repair time of defective items. Repair time is considered by $t_R = \beta(F_1 + F_2)TD/R + t_T = \beta FTD/R + t_T$. The entire cost for repair store is given as $s_r + 2A + \beta FTD(c_{lm} + 2c_t + h_s t_R)$. h_r is the holding cost of rework item. If h is taken as original unit holding cost then h < h_r. The duration of cycle time is distributed into four parts. The individual part is indicated by F_i and it exists in the interval of [0,1]. The positive inventory level is taken as FT, and (1-F)T is the period of the time where shortage appears. F is equivalent to F_1 and F_2, whereas (1-F) is equivalent to F3 and F4. It is assumed that when repaired items are received, shortages still remain in the inventory system. In the begining of the cycle, FTD is the inventory level. The items are screened at rate x and inspection time is given by $t_s = FTD/x$. After inspection, it has been established that β fraction volume of items is imperfect. These defective items $\beta FTD = F_2TD$ are sent to the repair shop. If shortage amount is given by (1-F)TD then, $\gamma(1-F)TD$ is the required shortage backordered and $(1-\gamma)(1-F)TD$ is the amount of lost sales. Figure 2 shows the in-depth inventory behavior of the developed system.

Setup cost: In the begining of cycle T, buyer suffers a one-time ordering cost per unit time (OC), where O is given as fixed ordered cost and it is expressed in (1).

$$OC = \frac{O}{T} \tag{1}$$

Screening cost: The total screening cost of the entire lot is represented by IC as in (2).

$$IC = C_s FD \tag{2}$$

where C_s the screening cost of one unit, F is the period of positive inventory time, and D is the demand rate.

Holding cost: (3) defines the holding cost per time unit,

$$HC = h\left[\frac{(1-\beta)^2F^2TD}{2} + \frac{\beta T(FD)^2}{x}\right] \tag{3}$$

Rework cost: If m margin is demanded as repair expense per unit, then repair cost (c_R) for a single unit is stated as in (4).

$$c_R(FTD) = (1+m)\left[\frac{s_r+2A}{\beta FTD} + c_{lm} + 2c_t + h_s t_R\right] \tag{4}$$

When the repaired item is arrived at buyer's system, shortages still exist, and the shortage level is $(F_2+F_3)TD$. Even after its addition, inventory level tends to be negative and it remains untill the cycle gets completed. Now the shortage becomes $(F_2+F_3+F_4)TD$. The order quantity in given cycle is then expressed as, $Q=(F_1+F_2)Td+\beta(F_3+F_4)Td = FTD+\beta(1-F)Td$

Shortage cost: If π is taken as backordered cost, l is the lost cost for sales and γ is a proportion of backordered demand. Then the shortage cost per unit time is given as in (5).

$$SC = \pi\frac{(F_2+F_3+F_4)(F_3+F_4)\gamma TD}{2} + l(1-\gamma)(F_3+F_4)D \tag{5}$$

Goodwill penalty cost: If g is the unit return cost, v is unit penalty cost after goodwill loss, and w percentage of imperfect items distributed to the consumers, then goodwill penalty cost is given in (6).

$$GWC = (v+g)wFD \tag{6}$$

Interest charged & interest income earned: In trade-credit-policy, larger permissible payment period than lead time brings interest income to the buyer. In the reverse case, when allowed period is shorter than lead time, it gives extra opportuity cost and interest income. At the same time, the supplier is also benefitted by the same. Thus the supplier's model includes two scenarios. Based on permissible payment period X and lead time length, two possible conditions are framed.

Condition 1: If lead time T $\leq$ X (permissible payment period) provided by supplier to the buyer, then only interest income is received and interest charged in this condition is zero. It is shown in figure 3, and is denoted in (7).

$$Interest\ income = PI_e\left[DX - \frac{TD}{2}\right] \tag{7}$$

Condition 2: If lead time T < X (first permissible payment time) and $\leq$ Y (second permissible payment time) provided by the supplier to the buyer, then interest cost is earned and charged. Figure 4 demonstrates the specified condition and is given in (8) and (9).

$$Interest\ income = PI_e\frac{(DX)^2}{2DT} \tag{8}$$

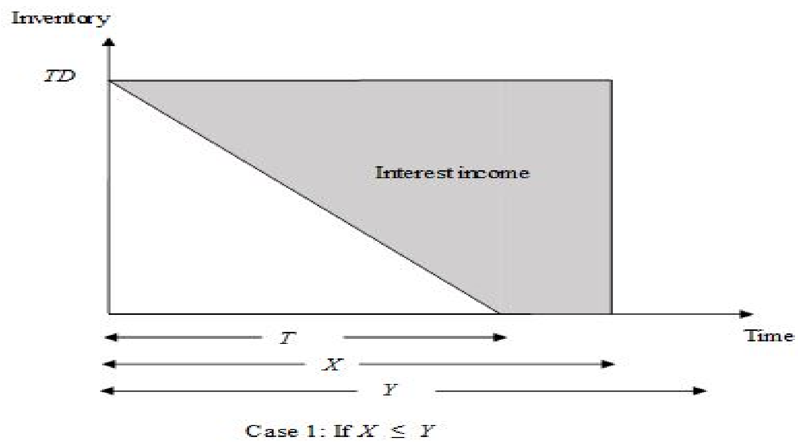

Figure 3.Earned and charged interest for T $\leq$ X

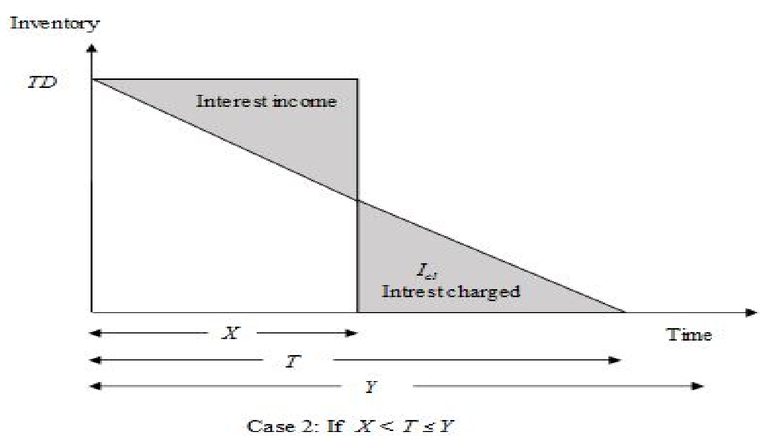

Figure 4.Earned and charged interest for X < T $\leq$ Y

$$Interest\ charged = C_u I_{c1} \frac{(TD - DX)^2}{2DT} \tag{9}$$

Condition 3: This is a special situation in which additional interest is charged to the buyer if he/she does not pay the required payment in first permissible time. In this scenario, lead time T is greater than second permissible payment period Y provided by the supplier to the buyer. Figure 5 shows the graphical illustration of this condition. The equation for interest income and interest charged is given in (10) and (11).

$$Interest\ income = PI_e \frac{(DX)^2}{2TD} \tag{10}$$

$$Interest\ charged = C_u I_{c2} \frac{(TD - DY)^2}{2TD} + C_u I_{c1} \frac{D}{T}(YT - Y^2 - XT + XY) + C_u I_{c1} \frac{D(X-Y)^2}{2D} \tag{11}$$

Total profit function:

Total profit = $Sellingprice - \Big[Ordering\ cost + Product\ cost + Screening\ cost + Holding\ cost + Rework\ cost + Shortage\ cost + Lost\ scale\ cost + Good\ will\ penalitycost + Interest\ earned - Interest\ charged \Big]$

According to these conditions of multi-delay-in-payments, three cases are established and the total profit function for these cases can be framed as,

Case 1: Total profit $(TP_1) if\ T \leq X$,

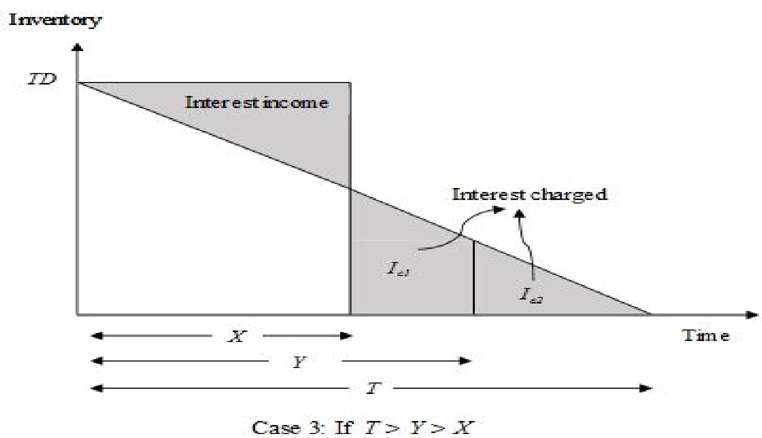

Figure 5.Earned and charged interest for T < Y ≤ X

$$TP(F,T) = PD(F+\gamma(1-F)) - \left[\frac{O}{T} + C_u(FD+\gamma(1-F)D) + C_s(F_1+F_2)D + h\left[\frac{(1-\beta)^2F^2TD}{2} + \frac{\beta T(FD)^2}{x}\right] + \beta FD(1+m)\left[\frac{s_r+2A}{\beta FTD} + c_{lm} + 2c_t + h_s\left(\frac{\beta FTD}{R} + t_T\right)\right] + \pi\frac{(F_2+F_3+F_4)(F_3+F_4)\gamma TD}{2} + l(1-\gamma)(F_3+F_4)D + (u+g)wFD + PI_e\left[DX - \frac{TD}{2}\right]\right] \tag{12}$$

Case 2: Total profit $(TP_2) if\ X < T \leq Y$,

$$TP(F,T) = PD(F+\gamma(1-F)) - \left[\frac{O}{T} + C_u(FD+\gamma(1-F)D) + C_s(F_1+F_2)D + h\left[\frac{(1-\beta)^2F^2TD}{2} + \frac{\beta T(FD)^2}{x}\right] + \beta FD(1+m)\left[\frac{s_r+2A}{\beta FTD} + c_{lm} + 2c_t + h_s\left(\frac{\beta FTD}{R} + t_T\right)\right] + \pi\frac{(F_2+F_3+F_4)(F_3+F_4)\gamma TD}{2} + l(1-\gamma)(F_3+F_4)D + (u+g)wFD + PI_e\frac{(DX)^2}{2DT} - C_uI_{c1}\frac{(TD-DX)^2}{2DT}\right] \tag{13}$$

Case 3: Total profit (TP_3) if T > Y > X,

$$TP(F,T) = PD(F+\gamma(1-F)) \left[\frac{O}{T} + C_u(FD+\gamma(1-F)D) + C_s(F_1+F_2)D + h\left[\frac{(1-\beta)^2F^2TD}{2} + \frac{\beta T(FD)^2}{x}\right] + \beta FD(1+m)\left[\frac{s_r+2A}{\beta FTD} + c_{lm} + 2c_t + h_s\left(\frac{\beta FTD}{R} + t_T\right)\right] + \pi\frac{(F_2+F_3+F_4)(F_3+F_4)\gamma TD}{2} + l(1-\gamma)(F_3+F_4)D + (u+g)wFD + PI_e\frac{(DX)^2}{2DT} - C_uI_{c2}\frac{(TD-DY)^2}{2DT} + C_uI_{c1}\frac{D}{T}(YT - Y^2 - XT + XY) + C_uI_{c1}\frac{D(X-Y)^2}{2D}\right] \tag{14}$$

where P is given as selling price of one unit, C_u is expressed as product cost of one unit, I_e is given as interest earned, I_{c1} is given as interest charged for permissible delay period X and I_{c2} is the interest charged for permissible delay period Y.

4.1. Optimal values of F and T for case 1

The profit function for case 1 in (12) can be re-written as: $F+\gamma(1-F)=1-1+F+\gamma(1-F)=1-(1-\gamma)(1-F)$ and $F_2+F_3+F_4=1-F_1=1-(1-\beta)F$ and by applying $C_z=(P+g-c_u)$ the total profit function is obtained as given in (15).

$$TP(F,T)=(P-C_u)D)-C_z(1-\gamma)D$$
$$-\left[\frac{O}{T}+C_sFD+h\left[\frac{(1-\beta)^2F^2TD}{2}+\frac{\beta T(FD)^2}{x}\right]\beta FD(1+m)\left[\frac{s_r+2A}{\beta FTD}+c_{lm}+2c_t+h_s\left(\frac{\beta FTD}{R}+t_T\right)\right]\right.$$
$$\left.+(1-(1-\beta)F)(1-F)\frac{\pi\gamma TD}{2}-C_{z(1-\gamma)}DF+(u+g)wFD+PI_e\left[DX-\frac{TD}{2}\right]\right] \tag{15}$$

The optimal value for F and T can be calculated as in (15) and (16) by using algebraic approach.

$$\pi\gamma(2-\beta)T+2C_z(1-\gamma)-2C_s-2\beta(1+m)$$
$$F^*=\frac{(c_{lm}+2c_t+h_st_T}{4\left[\frac{(1+m)h_s\beta^2D}{R}+\frac{(1-\beta)^2h}{2}+\frac{\beta hD}{x}+\frac{\pi\gamma(1-\beta)}{2}\right]T} \tag{16}$$

$$T^*=\sqrt{\frac{4(O+(1+m)(s_r+2A))\left(\frac{(1+m)h_s\beta^2D}{R}+\frac{(1-\beta)^2h}{2}+\frac{\beta hD}{x}+\frac{\pi\gamma(1-\beta)}{2}\right)-D\left(C_s-C_z(1-\gamma)+\beta(1+m)^2(c_{lm}+2c_t+h_st_T)\right)^2}{\left(2\pi\gamma+\frac{PI_e}{2}\right)\left(\frac{(1+m)h_s\beta^2D^2}{R}+\frac{(1-\beta)^2hD}{2}+\frac{\beta hD^2}{x}+\frac{\pi\gamma D\beta^2}{8}\right)}} \tag{17}$$

4.2. Optimal values of F and T for case 2

The profit function for case 1 in (13) can be redefined as: $F+\gamma(1-F)=1-1+F+\gamma(1-F)=1-(1-\gamma)(1-F)$ and $F_2+F_3+F_4=1-F_1=1-(1-\beta)F$ and by applying $C_z=(P+g-c_u)$ the total profit function is given as in (18).

$$TP(F,T)=(p-C_u)D)-C_z(1-\gamma)D$$
$$-\left[\frac{O}{T}+C_sFD+h\left[\frac{(1-\beta)^2F^2TD}{2}+\frac{\beta T(FD)^2}{x}\right]+\beta FD(1+m)\left[\frac{s_r+2A}{\beta FTD}+c_{lm}+2c_t+h_s\left(\frac{\beta FTD}{R}+t_T\right)\right]\right.$$
$$\left.+(1-(1-\beta)F)(1-F)\frac{\pi\gamma TD}{2}-C_z(1-\gamma)DF+(u+g)wFD+PI_e\frac{(DX)^2}{2TD}=C_uI_{cl}\frac{(TD-Dx)^2}{2TD}\right] \tag{18}$$

The optimal value for F and T can be calculated by using algebraic approach.

$$\pi\gamma(2-\beta)T+2C_z(1-\gamma)-2C_s-2\beta(1+m)$$
$$F^*=\frac{(c_{lm}+2c_t+h_st_T}{4\left[\frac{(1+m)h_s\beta^2D}{R}+\frac{(1-\beta)^2h}{2}+\frac{\beta hD}{x}+\frac{\pi\gamma(1-\beta)}{2}\right]T} \tag{19}$$

$$T^*=\sqrt{\frac{4\left(O+(1+m)(s_r+2A)+\frac{C_uI_{cl}DX^2}{2}-\frac{PI_eDX^2}{2}\right)\left(\frac{(1+m)h_s\beta^2D}{R}+\frac{(1-\beta)^2h}{2}+\frac{\beta hD}{x}+\frac{\pi\gamma(1-\beta)}{2}\right)-D\left(C_s-C_z(1-\gamma)+\beta(1+m)^2(c_{lm}+2c_t+h_st_T)\right)^2}{\left(2\pi\gamma+\frac{C_rI_{cl}}{2}\right)\left(\frac{(1+m)h_s\beta^2D^2}{R}+\frac{(1-\beta)^2hD}{2}+\frac{\beta hD^2}{x}+\frac{\pi\gamma D\beta^2}{8}\right)}} \tag{20}$$

4.3. Optimal values of F and T for case 3

The profit function for case 1 in (14) can be re-written as: $F+\gamma(1-F)=1-1+F+\gamma(1-F)=1-(1-\gamma)(1-F)$ and $F_2+F_3+F_4=1-F_1=1-(1-\beta)F$ and by applying $C_z=(P+g-c_u)$, the total profit function is obtained as in (21).

$$
\begin{aligned}
TP(F,T) = (p-C_u)D) - C_z(1-\gamma)D - \Bigg[\frac{O}{T} + C_sFD + h\left[\frac{(1-\beta)^2F^2TD}{2} + \right. \\
\frac{\beta T(FD)^2}{x} + \beta FD(1+m)\left[\frac{s_r+2A}{\beta FTD} + c_{lm} + 2c_t + h_s\left(\frac{\beta FTD}{R} + t_T\right)\right] + \\
(1-(1-\beta)F)(1-F)\frac{\pi\gamma TD}{2} - C_z(1-\gamma)DF + (u+g)wFD + PI_e\frac{(DX)^2}{2TD} + \\
C_uI_{c2}\frac{(TD-DY)^2}{2TD} + C_uI_{cl}\frac{D}{T}(YT-Y^2-XT+XY)C_uI_{cl}\frac{D(X-Y)^2}{2D}\Bigg]
\end{aligned}
\tag{21}
$$

Algebraic approach is used to calculate the optimal value for F and T.

$$
F^* = \frac{\pi\gamma(2-\beta)T + 2C_z(1-\gamma) - 2C_s - 2\beta(1+m)(c_{lm} + 2c_t + h_st_T}{4\left[\frac{(1+m)h_s\beta^2D}{R} + \frac{(1-\beta)^2h}{2} + \frac{\beta hD}{x} + \frac{\pi\gamma(1-\beta)}{2}\right]T}
\tag{22}
$$

$$
T^* = \sqrt{\frac{4\left(O+(1+m)(s_r+2A)+\frac{C_uI_{c2}DY^2}{2}-\frac{PI_eDX^2}{2}-C_uI_{cl}DY^2+C_uI_{cl}DXY+\frac{C_uI_{cl}D(X-Y)^2}{2}\right)\left(\frac{(1+m)h_s\beta^2D}{R}+\frac{(1-\beta)^2h}{2}+\frac{\beta hD}{x}+\frac{\pi\gamma(1-\beta)}{2}\right)-D\left(C_s-C_z(1-\gamma)+\beta(1+m)^2(c_{lm}+2c_t+h_st_T)\right)^2}{\left(2\pi\gamma+\frac{C_rI_{c2}}{2}\right)\left(\frac{(1+m)h_s\beta^2D^2}{R}+\frac{(1-\beta)^2hD}{2}+\frac{\beta hD^2}{x}+\frac{\pi\gamma D\beta^2}{8}\right)}}
\tag{23}
$$

5. NUMERICAL EXAMPLE

Model validation is presented in this section by describing the numerical solution of the given example. The optimal results for the total profit and decision variables for all three cases are given in table 1. The numerical data is given in table 2 [26], with additional parameters of l, u, w, X, Y, I_e, I_{c1}, I_{c2}.

Table 1.Optimum total profit for three cases with different values of decision variables

Scenario	T (year)	F (%)	Q (units)	TP ($)
Case 1	0.052	0.67	2600	1204125
Case 2	0.084	0.72	4200	1201175
Case 3	0.126	0.75	6300	1194535

5.1. Results

The result illustrates that optimal solution for case 1 occurs when T is 0.052 year, the percentage of this cycle time having positive inventory level with F = 0.67 %, the ordered quantity for this scenario is 2600 units and the entire profit is 1204125 $. For case 1, cycle time T is lesser or equal to the allowable payment period X given by the supplier to the buyer. In case 2, the optimal value is attained with cycle time period with T = 0.084 year, the percentage of this cycle time having positive inventory level with F = 0.72 %, the ordered quantity for this case is 4200 units, and total profit is 1201175 $. In this scenario, cycle time T is higher than first allowable payment period X and lesser or equal to second allowable payment period Y given by the supplier to the buyer. And for case 3, the optimal solution is attained when cycle time period T = 0.126 year, the percentage of this cycle time having positive inventory level with F = 0.75 %, the ordered quantity for this case is 6300 units and the entire profit is 1194535 $. The cycle time T is higher than both allowable payment periods X and Y given by the supplier to the buyer. From results, it is shown that maximum profit is obtained in case 3 and minimum profit is achieved in case 1. The results show that total profit is lower if delay-in-payment period given by the supplier to the buyer is higher or equal to the cycle time and annual profit is higher, if delay-in-payment period given by the supplier to

Table 2.Data for numerical example

Parameter	Symbol	Value	Units
Demand rate	D	5000	units/ year
Screening rate	x	175200	units/ year
Repaired rate	R	50000	units/ year
Ordering coste	O	100	$/order
Purchase cost	C_u	25	$/unit
Selling price	P	50	$/unit
Holding cost of the perfect product	h	5	$/unit/year
Holding cost at the repair store	h_s	4	$/unit/year
Holding cost of rework product	h_r	6	$/unit/year
Inspection cost	C_s	0.5	$/unit
Backorder cost	π	20	$/unit/year
Lost sales cost	l	0.5	$/unit/year
Fraction of backordered demand	γ	97%	percent
Fixed setup cost of repair store	s_r	100	$/setup
Fixed cost (transportation)	A	200	$/trip
Transpiration cost of unit	c_t	2	0.4 $/unit
Labour and material cost of unit	c_{lm}	5	$/unit
Transport time	t_T	2/220	year
Mark-up percentage	m	20%	percent
Percentage of imperfect items	β	0.04%	percent
Penalty cost from goodwill loss	u	15	$/unit
Return cost	g	3	$/unit
Percentage of imperfect items returned	w	0.02%	percent
First permissible delay period	X	30	days
Second permissible delay period	Y	45	days
Interest earned	I_e	12%	percent
Interest charged	I_{c1}	13%	percent
Interest charged	I_{c2}	20%	percent

the buyer is lesser than the cycle time. Also, the interest earned due to this short-term financial aid by selling these units helps the buyer in profit increment. If F has the value exactly as one, then no shortages occur. On the other hand, if F has the value equal to zero, loss occurs in all demand. Additional analysis on total profit by making a change in different parameters are presented in next section.

6. SENSITIVITY ANALYSIS

This section shows the effect on the total profit for all cases by changing several key parameters $h, \pi, \gamma, I_e, I_{c1}$, I_{c2}, X and Y respectively. The analysis of the given parameters for the given model for different scenarios is presented in table 3. Figures 6-8 show the graphical representation of the impact due to some key parameters on total profit.

The following outcomes are carried out:

- Variation in holding cost h does not have any effect in total profit for each case. Moreover, percentage increase in h by 25% and 50% creates a fall in TP of 0.11% and 0.06% for case 1, cuts TP 0.10% and 0.21% for case 2 and falls TP 0.17% and 0.34% for case 3. Also, decrement in the value of h by 25% and 50%, the total profit upturns by 0.06% and 0.06% for case 1, by 0.10% and 0.21% for case 2 and by 0.17% and 0.34% for case 3. It shows that it exists in an equilibrium position. Furthermore, it illustrates that TP is parallel sensitive with a negative and positive variation of parameter h.
- Change in parameter value in backorder cost π makes a minor change in total profit for all cases. However, the increment in the parameter π by 25% and 50% causes a reduction in total profit for each case. On the other hand, decrement in the parameter π by 25% and 50% upturns the total profit. It is to state that it holds

Table 3.Sensitivity analysis of the key parameters

Parameter	% change	% change TP (T, F)		
		Case 1	Case 2	Case 3
h	+50	-0.11	-0.21	-0.34
	+25	-0.06	-0.10	-0.17
	-25	+0.06	+0.10	+0.17
	-50	+0.11	+0.21	+0.34
π	+50	-0.12	-0.14	-0.17
	+25	-0.06	-0.07	-0.09
	-25	+0.06	+0.07	+0.09
	-50	+0.12	+0.14	+0.17
γ	+50	+17.16	+14.63	+13.14
	+25	+8.58	+7.31	+6.57
	-25	-8.936	-7.62	-6.84
	-50	-17.51	-14.93	-13.42
I_e	+50	+0.71	+0.51	+0.29
	+25	+0.36	+0.26	+0.15
	-25	-0.35	-0.26	-0.15
	-50	-0.71	-0.51	-0.29
I_{c1}	+50	N.A	+0.51	+0.29
	+25	N.A	+0.26	+0.15
	-25	N.A	-0.26	-0.15
	-50	N.A	-0.51	-0.29
I_{c2}	+50	N.A	N.A	+0.29
	+25	N.A	N.A	+0.15
	-25	N.A	N.A	-0.15
	-50	N.A	N.A	-0.29
X	+50	+1.03	-0.36	N.A
	+25	+0.52	+0.24	N.A
	-25	-0.52	-0.31	N.A
	-50	-1.09	-0.77	N.A
Y	+50	N.A	-0.36	-0.04
	+25	N.A	+0.24	+0.003
	-25	N.A	-0.31	-0.03
	-50	N.A	-0.77	-0.23

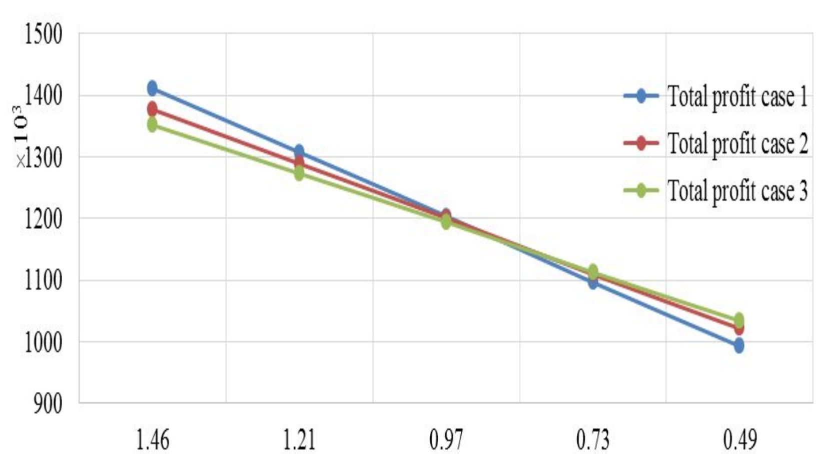

Figure 6.Impact of changing the fraction of backorered demand (γ) on the total profit for each scenario

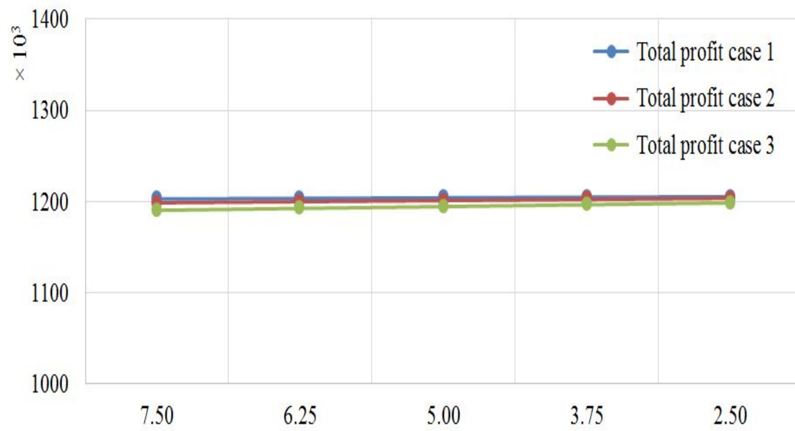

Figure 7.Impact of changing holding cost (h) on the total profit for each scenario

an equilibrium state. Also, both negative and positive change of parameter π causes a change in total profit.

- The outcome of parametric variation in backorder demand γ percentage creates huge impact in total profit for each case. In addition, γ increases by 25 and 50% results in increment for each case. Likewise, decrement in the value of parameter γ by 25% and 50% decreases the total profit in all cases and is postioned in an equilibrium state. It also indicates total profit is sensitive to both negative and positive changes of parameter γ.
- A small change occurs in total profit for all cases due to the variation in interest earned per dollar per year I_e. In addition, if the value of I_e is enlarged by 25% and 50%, then total profit is also increased for all cases. Likewise, decrement in the parameter I_e by 25% and 50% causes a decline in total profit. The result demonstrates that total profit is proportional to interest earned, and moreover it also lies in an equilibrium state.

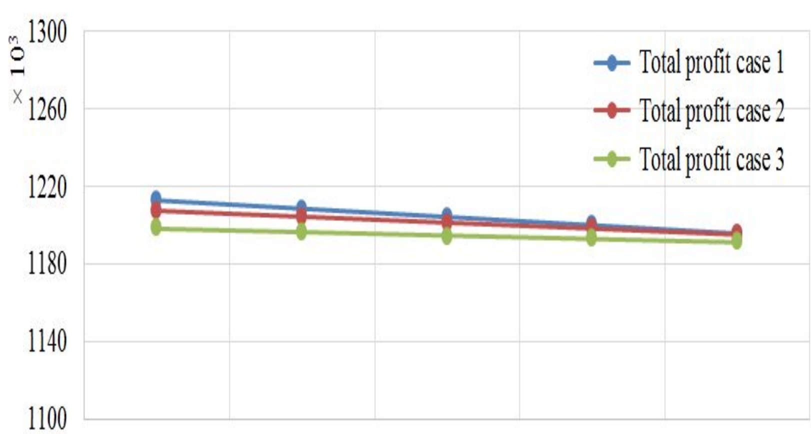

Figure 8.Impact of changing percentage of interest earned (I_e) on the total profit for each scenario.

- The parametric variation in the value of interest charged I_{c1} and I_{c2} are not applicable to case 1, also, I_{c2} is not applicable to case 2. In case 2 and case 3, I_{c1} makes a small change in total profit. Also, if the value of I_{c1} is expanded by 25% and 50% the total profit rises for both cases. On the contrary, shrinkage in the value of parameter I_{c1} by 25% and 50%, drops the annual total profit. Case 3 shows similar pattern for parameter I_{c2}. An equilibrium state also exists in their patterns.
- First delay-in-payment period X marginally varies the annual total profit of first two cases. The incremental adjustment in this parameter by 25% and 50% makes an increase in total profit for case 1, but on another hand with an increase of 50% in this parameter, the total profit is reduced for case 2. For 25% increment in given

parameter, case 2 shows the same pattern as of case 1. In contrast, a drop in parameter X by 25% and 50%, makes a decrease in total profit for these cases.

- Second delay-in-payment period Y causes a change in overall profit for the second and third case. The case 1 has no concern with it. The incremental change for this parameter by 25% makes an increase in total profit for these cases. On the other hand, by decreasing 25% and 50% and by increasing 50%, the total profit is decreased in these cases.

7. MANAGERIAL INSIGHTS

This study establishes strategic and thoughtful insights for managers who deal with defective products, reparation of the imperfect products and multi-trade-credit policy. The selection of situation in which cycle time is greater than both permissible delay payment time offered by the supplier to the buyer results in maximum total profit. The managers have to create a conclusion on the perceptive of cycle time and a portion of the duration with positive inventory. These factors must be adjusted in a direction to attain a win-win situation. This work also demonstrates in-depth sensitivity analysis on total profit by fluctuating the main input variables such as backorder demand, unit holding cost and interest earned and interest paid. Furthermore, this research shows a pathway to handle imperfect products if the seller is located miles away. These imperfect products have value and can be part of the total profit. To get more revenue, policy-makers should emphasis on required cycle time, an element of backordered demand and permissible delay-in-payment strategies. In order to increase the sales, the industrial managers have to decide the delay-in-payment that is required to cope with the reduction of the inventory in stock. To maximize the profit, the industrial managers have to give more attention to monitor the key considerations, by systematically understanding the relation as presented in sensitivity analysis. In short, the present analysis provides a systematical guidelines for industrial policy-makers to attain optimal results.

8. CONCLUSIONS

This research considers the inventory model with synergic effects of reparation for defective items with incorporation of backordering and multi-trade-credit-policy. In this study, a situation is generated in which supplier is to be found globally. The manufacturing system of the supplier may not produce all desired quality items. And it is possible that received lot by the buyer have some fraction of imperfect items. It is impossible for the buyer to exchange these imperfect items instantly. The imperfect units have a worth and can be reworked in smaller time, to save cost as compared to high exchange cost with the supplier. In such situation, this work helps to extend the inventory model with defective items, repair these products in the local repair store to reduce environmental concern and partial backordering. The imperfect units can be put out of system as soon as inspection procedure is completed and dispatched to a local repair shop in a single lot. After rework, the items are arrived back to the buyer. Shortages still occur in the system when the rework products are arrived. The inventory system also synchronizes with trade-credit strategy. The multi-delay-in-payment work increases the entire sale due to the effect of short-term financial investment. The optimal solution in accordance with different cases of cycle time under permissible delay-in-payment is concluded. The non-derivative approach is used to accomplish an optimal solution of the developed model. This method has shown confidently in earlier inventory models. Lastly, a numerical example is demonstratrd to express this inventory model and sensitivity analysis of key input parameters is done.

In this study, we make an effort to exploit structural properties in the total profit function by making a payment under the multi-trade-credit-policy and to mark theoretical outcomes. The co-operative understanding among businesses parties is used to develop the optimal total profit. The developed work has impacts on firm short-term financing. The managers should adopt the use of settlement policy for profit growth. If the original unit holding cost of flawless items is higher than the second permissible payment period, it is not feasible to attain the optimal solution. This proposed model helps in enhancing the performance by monitoring cycle time. The future research can be done in multiple directions by focussing on the limitation of this model, i.e. with the multi-item case. The implication of multi-echelon supply chain in these models is also a promising opportunity for future study.

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