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RESEARCH ARTICLE

A Dark Energy Model in the Presence of Scalar Meson Fields in General Relativity

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ABSTRACT

The present article considers a spatially homogeneous and anisotropic Bianchi type V space time under scalar meson fields and anisotropic dark energy in general relativity. Solving of the general relativity field relations using (i) special law of variation for Hubble's parameter results in constant value of deceleration parameter and (ii) the shear scalar of the space-time is proportional to the expansion scalar. This exact solution represents a dark energy cosmological model under zero mass scalar fields. The dynamical and geometrical parameters of the model are determined and their physical significance with reference to our model is discussed.
Keywords: Zero mass scalar fields, Bianchi-V model, Hubble's parameter, General relativity, Dark energy model.

1. INTRODUCTION

Renewed interest in Dark Energy (DE) models of the universe is owing to the fact that the universe is accelerating which is driven by a negative exotic pressure dubbed as dark energy [1-3]. However, DE is yet to be revealed further. Among the various parameters defined by many researchers, the cosmological constant ($\wedge$) with the Equation of State (EoS) parameter, ω = -1 is considered to be the most apparent theoretical candidate of DE; but is affected by fine tuning and cosmic coincidence problems [4, 5]. Hence viable alternatives for the dynamic DE have been suggested. Noteworthy among them are i) scalar field or quintessence DE models [5], (ii) phantom (ghost) model [6], (iii) k-essence model [7], (iv) quintom [8], (v) Chaplygin gas [9] and (vi) holographic DE models [10]. Here scalar field DE models are mainly focused, and for this purpose we consider scalar meson fields in general relativity, where the Einstein field relations under scalar meson fields are briefly discussed.

Several researchers concentrate on scalar meson fields in general relativity due to their physical significance in particle physics. They refer to matter field with spin less quanta and is classified into: zero rest and massive scalar fields. Both of them represent long and short range interactions respectively. [12] Einstein field relations under anisotropic DE fluid and scalar meson fields are given in (1),

$$R_{ij}-\frac{1}{2}g_{ij}R=-(T_{ij}^{(m)}+T_{ij}^{(s)}) \tag{1}$$

where $T_{ij}^{(m)}$ and $T_{ij}^{(s)}$ are the energy momentum tensors of the anisotropic DE fluid and zero mass scalar field as given in (2) and (3),

$$T_{ij}^{(m)}=(\rho_{\wedge}+p_{\wedge})u_iu_j-g_{ij}p_{\wedge} \tag{2}$$

$$T_{ij}^{(s)}=V_{,i}V_{,j}-\frac{1}{2}g_{ij}(V_{,k}V^{,k}) \tag{3}$$

Here $\rho_{\wedge}$ and $p_{\wedge}$ are the energy density and pressure of the DE fluid respectively and V is the real scalar field satisfying the Klein –Gordon equation (4).

$$g^{ij}V_{,ij}=0 \tag{4}$$

where a semi colon denotes covariant derivative and a comma indicates ordinary derivative. Here V is called the

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mass less scalar meson field.

Some significant contributions of Bianchi models under scalar meson fields are mentioned. [13] has discussed some non-static solutions of Einstein field relations under viscous fluid and massive scalar field. [14] has obtained plane symmetric cosmological models with viscous fluid and massive scalar field as the source. [15] has discussed certain inhomogeneous cosmological models in Bianchi type-V space time filled with viscous fluid under a massive scalar field. [16] has studied massive scalar field in Bianchi type-I space-time. [17] has discussed Bianchi type-V cosmologies. Lyra [18] has elaborated studies under massive scalar field while [19] has obtained Kantowski-Sachs cosmological model under mass less scalar field. [20] has examined Bianchi type-III cosmological models in Lyra geometry under massive scalar field. [21] has discussed Kantowski–Sachs minimally interacting holographic DE model with linearly varying deceleration parameter in Saez–Ballester theory. [20-25] have discussed anisotropic DE models in scalar-tensor theories of gravitation.

Based on the literature studies, it is clear that the researches relating the spatially homogeneous and anisotropic Bianchi-V DE models under mass less scalar meson fields are not given importance, and that has motivated us to go ahead with this research work, Bianchi type-V anisotropic dark energy models in the presence of scalar meson fields. The plan of this work is as follows: Section 2 deals with the derivation of the basic field equations. In section 3, the solutions of the field equations and the corresponding models are described. Section 4 highlights the dynamical parameters and their physical significance which are widely significant in cosmology. Finally, the article is summarized and concluded in section 5.

2. METRIC AND FIELD EQUATIONS

Spatially homogeneous and anisotropic Bianchi type-V space-time is given in (5),

$$ds^2 = dt^2 - X^2dx^2 - Y^2e^{-2x}dy^2 - Z^2e^{-2x}dz^2 \tag{5}$$

where X,Y and Z are functions of cosmic time t. The energy- momentum tensor of anisotropic [26] dark energy fluid given by (2) can be parameterized as in (6),

$$T_{ij} = diag[1, \omega_\wedge, -(\omega_\wedge+\gamma), -(\omega_\wedge+\delta)]\rho_\wedge \tag{6}$$

where

$$\omega_\wedge = \frac{p_\wedge}{\rho_\wedge} \tag{7}$$

is the EoS parameter of DE. Here the skewness parameters γ and δ are the deviations from $\omega_\wedge$ along y and z-axes respectively.

Using co-moving coordinates Einstein equations (1) with the help of (3) and (6) for the metric (5) can explicitly be written as,

$$\frac{\dot{X}\dot{Y}}{XY} + \frac{\dot{Y}\dot{Z}}{YZ} + \frac{\dot{Z}\dot{X}}{ZX} - \frac{1}{X^2} - \frac{\dot{V}^2}{2} = \rho_\wedge \tag{8}$$

$$\frac{\ddot{Y}}{Y} + \frac{\ddot{Z}}{Z} + \frac{\dot{Y}\dot{Z}}{YZ} - \frac{1}{X^2} - \frac{\dot{V}^2}{2} = -\omega_\wedge\rho_\wedge \tag{9}$$

$$\frac{\ddot{X}}{X} + \frac{\ddot{Z}}{Z} + \frac{\dot{Z}\dot{X}}{ZX} - \frac{1}{X^2} - \frac{\dot{V}^2}{2} = -(\omega_\wedge+\gamma)\rho_\wedge \tag{10}$$

$$\frac{\ddot{X}}{X} + \frac{\ddot{Y}}{Y} + \frac{\dot{X}\dot{Y}}{XY} - \frac{1}{X^2} - \frac{\dot{V}^2}{2} = -(\omega_\wedge+\delta)\rho_\wedge \tag{11}$$

$$\frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z} + 2\frac{\dot{X}}{X} = 0 \tag{12}$$

and the Klein-Gordon equation for the metric (5) takes the form (13),

$$\ddot{V}+\dot{V}\left[\frac{\dot{X}}{X}+2\frac{\dot{Y}}{Y}\right]=0 \tag{13}$$

The conservation law for the energy momentum tensor of Dark energy yields (14),

$$\dot{\rho_\wedge}+(\frac{\dot{X}}{X}+\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z})(\rho_\wedge+p_\wedge)=0 \tag{14}$$

where an over head dot indicates derivative with respect to cosmic time t.The cosmological parameters are defined in (15) which would help in solving the above field equations.

The volume v is given in (15)

$$v=a^3=XYZ \tag{15}$$

The average Hubble parameter H is defined in (16),

$$H=\frac{\dot{a}}{a}=\frac{1}{3}(\frac{\dot{X}}{X}+\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z}) \tag{16}$$

The scalar expansion θ and the shear scalar $\sigma^2$are given in (17) and (18) respectively.

$$\theta=3H=\frac{\dot{X}}{X}+\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z} \tag{17}$$

$$\sigma^2=\frac{1}{2}\sigma^{ij}\sigma_{ij}=\frac{1}{2}\left[\left(\frac{\dot{X}}{X}\right)^2+\left(\frac{\dot{Y}}{Y}\right)^2+\left(\frac{\dot{Z}}{Z}\right)^2-\frac{\dot{X}}{X}\frac{\dot{Y}}{Y}-\frac{\dot{Y}}{Y}\frac{\dot{Z}}{Z}-\frac{\dot{Z}}{Z}\frac{\dot{X}}{X}\right] \tag{18}$$

The mean anisotropy parameter is defined as in (19).

$$A_h=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2 \tag{19}$$

3. SOLUTIONS AND THE MODELS

The field equations (8)-(14) are solved to obtain deterministic DE models under scalar meson fields. Integrating (12), we get as in (20).

$$X^2=k_1YZ \tag{20}$$

without any loss of generality, the constant of integration k_1 can be chosen as unity so that we have (21),

$$X^2=YZ \tag{21}$$

The set of field (9)-(11) together with (13) and (14) are a system of four independent equations (14), being the conservation equation, is consequence of the field equations in X,Y,Z,$\rho_\wedge,\omega,V,\delta,\gamma$. A determinate solution is found out using the physically significant conditions expressed below:

(i) the shear scalar of the space-time is proportional to the expansion scalar [27].

$$Y=Z^k \tag{22}$$

where k≠1 is a positive constant which preserves the anisotropy of the space time.

(ii) Special law of variation for Hubble's parameter [28] yielding constant deceleration parameter models of the universe is given in (23).

$$q=-\frac{a\ddot{a}}{\dot{a}^2}=constant \tag{23}$$

(24) is obtained by integrating (23).

$$a(t) = (ct+d)^{\frac{1}{1+q}} \tag{24}$$

where c_0, c and d are integration constants. (23) states that the condition for spatial expansion of the universe is 1+q>0.

Now, from (15), (21), (22) and (24) metric potentials are defined as in (25).

$$X = (ct+d)^{\frac{1}{1+q}}$$
$$Y = (ct+d)^{\frac{2k}{(k+1)(1+q)}}$$
$$Z = (ct+d)^{\frac{2}{(k+1)(1+q)}} \tag{25}$$

Using (13) and (25), the scalar field in the model is defined in (26).

$$V = V_0 \frac{(1+q)}{c(q-2)}(ct+d)^{\frac{q-2}{1+q}} \tag{26}$$

where V_0 is a integration constant. After a suitable choice of coordinates and constants (i.e. c=1, d=0) the metric (5) can be written as in (27)

$$ds^2 = dt^2 - t^{\frac{2}{1+q}}dx^2 - t^{\frac{4k}{(k+1)(1+q)}}e^{-2x}dy^2 - t^{\frac{4k}{(k+1)(1+q)}}e^{-2x}dz^2 \tag{27}$$

and the scalar field in the model takes the form as in (28),

$$V = V_0 \frac{(1+q)}{(q-2)}(t)^{\frac{q-2}{1+q}} \tag{28}$$

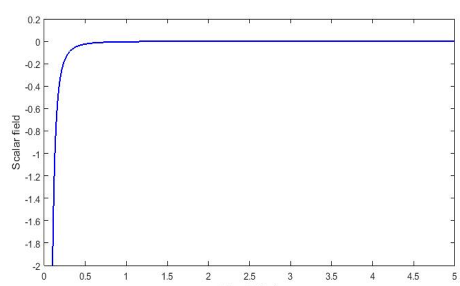

Figure 1.Plot of scalar field versus time for V_0=0.01, q=-0.2, k=0.7

4. DYNAMICAL PROPERTIES OF THE MODEL

Certain geometrical and physical quantities of the model given in (27), and their behaviour is computed. The model represents Bianchi type-V universe filled with DE and scalar meson fields. The geometrical and physical parameters which are important in cosmological models of the universe are as follows. The spatial volume, mean Hubble parameter, scalar expansion, shear scalar and the anisotropy parameter are defined in (29) to (33).

Spatial volume,

$$v = t^{\frac{3}{1+q}} \tag{29}$$

The mean Hubble parameter,

$$H = \frac{1}{(1+q)t} \tag{30}$$

The scalar expansion,

$$\theta = \frac{3}{(1+q)t} \tag{31}$$

The shear scalar,

$$\sigma^2 = \frac{3(k-1)^2}{[(k+1)(1+q)t]^2} \tag{32}$$

The anisotropy parameter,

$$A_h = \frac{2}{3}\left(\frac{k-1}{k+1}\right)^2 \tag{33}$$

Using (27) and (28) in (8), the DE density in the universe (29) is obtained as in (36).

$$\rho_\wedge = \frac{2(k^2+4k+2)}{[(k+1)(1+q)t]^2} - t^{\frac{-2}{1+q}} - \frac{V_0^2}{2}t^{-6} \tag{34}$$

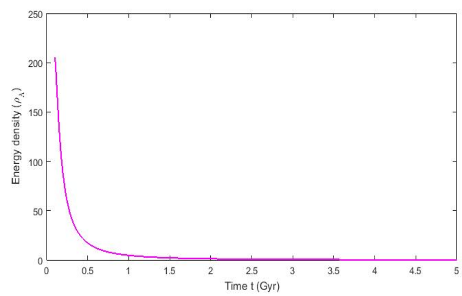

Figure 2.Plot of energy density versus time t for V_0=0.01, q=-0.2, k=0.7

From (9), (27), (28), the EoS parameter is resulted as in (35),

$$\omega_\wedge = -\left[\frac{8(k^2+k+1) - 4(1+q)(k+1)^2 - [(k+1)(1+q)]^2[2t^{\frac{q}{1+q}} - V_0^2t^{-4}]}{4(k^2+k+2)[(k+1)(1+q)]^2[2t^{\frac{q}{1+q}} - V_0^2t^{-4}]}\right] \tag{35}$$

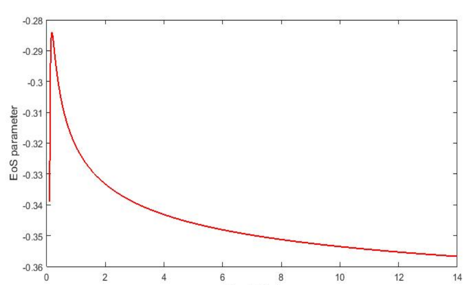

Figure 3.Plot of EoS parameter versus time t for V_0=0.01, q=-0.2, k=0.7

From (9), (10), (11), (27) and (34) we obtain the skewness parameters as in (36) and (37).

$$\gamma = \frac{2(k-1)[(k+1)(1+q)+3k]}{4(k^2+4k+2) - [(k+1)(1+q)]^2[2t^{\frac{q}{1+q}} + V_0^2t^{-4}]} \tag{36}$$

$$\delta = \frac{2(k+1)(k-1)(q-2)}{4(k^2+4k+2) - [(k+1)(1+q)]^2[2t^{\frac{q}{1+q}} + V_0^2t^{-4}]} \tag{37}$$

The following is the physical behavior of the universe model. The spatial volume of the universe increases with time. This shows the spatial expansion of the universe. The model is free from initial singularity. At the initial epoch, the quantities H, θ, σ^2, δ, γ and V tend to infinity and as t approaches infinity, they approach zero. .

From graphical representation, figure 1 shows that the scalar field in the model varies in the negative region and vanishes at late times. Figure 2 depicts the variation of DE density with time which shows that it decreases with time and vanishes for large t. It may be observed from figure 3 that the EoS parameter always varies in the quintessence region ($-1<\omega_\wedge<\frac{-1}{3}$) and hence quintessence model of the universe is obtained.
It may be observed that when k=1,

$$A_{h=0,\ \sigma^2=0,\ \gamma=0=\delta} \tag{38}$$

This shows that at late times, the model attains isotropy and becomes shear free. Also, the skewness of the model vanishes. This is similar to the present day observation of the universe.

Cosmic jerk parameter, j is used in cosmology for describing models closer to $\wedge$CDM. It is the 3^{rd} derivative of the scale factor in accordance with the cosmic time [29].

$$j(t) = \frac{1}{H^2}\frac{\dddot{a}}{a} = q + 2q^2 - \frac{\dot{q}}{H} = constant \tag{39}$$

The transition of the universe takes place from decelerated to the accelerated phase for models with positive value of jerk parameter and negative value of deceleration parameter. The jerk parameter for $\wedge$CDM model have a constant jerk, j=1.

5. SUMMARY AND CONCLUSIONS

The role of scalar fields is vital in the discussion of dark energy models since they represent quintessence models. There are several DE models in literature corresponding to Brans-Dicke [30] scalar fields and Saez-Ballester [31] scalar fields investigated by several authors. In this paper, we have discussed DE models in the presence of scalar meson fields in Bianchi type-V space time. Graphical representation is included to increase the understandability of the scalar field, DE density and EoS parameter of the model. It is observed that the scalar field in the model varies in the negative region and vanishes at late times. The energy density of DE decreases with time and vanishes for large t. The EoS parameter always varies in the quintessence region and hence the quintessence model of the universe is obtained. Observations of modern cosmology reveal that our universe is going through an accelerated expansion and will end up in a big rip in future. The discussion of our model supports this concept and will help for a better understanding of the late time acceleration of the universe and will throw a better light on the explanation of the mysterious concept of dark energy.

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