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RESEARCH ARTICLE

# An Algorithm to find Definite Integrals using Simpson Rule

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**ABSTRACT**

An algorithmic approach to find definite single and double integrals using Simpson's $\frac{1}{3}-$ rule is presented in this paper. This work will ensure the practical need of easy way of calculations with less computation time (run time) and storage space to engineers and scientists. It is observed that there is an opening to design a Simpson method which is different from the traditional Simpson method available in the literature for the numerical solution of ordinary differential equations. Numerical experiments were performed to show the validity of the algorithm.
**Keywords:** Simpson rule, Single and double integrals, Newton's interpolation, Less computation time, Storage space.

## 1. INTRODUCTION

In real time situations, scientists and engineers come across various practical difficulties in using both differential and integral equations as mathematical model for time dependent problems [1-4]. Many solutions to these problems do exist but with complicated steps it is further difficult to extend their ideas to higher dimensional problems. To make it easy, in this paper, a new approach of finding single and double definite integrals using Simpson rule are presented. In section 2, a definite single integral is derived and in section 3, a definite double integral is derived using Simpson rule. In section 4, a pictorial form of algorithm is given in two different forms for double integrals using Simpson rule and in section 5, numerical experiments are provided to show the performance of the single and double integrals using pictorial form of algorithm for Simpson's $\frac{1}{3}-$ rule. The analysis is carried out by means of equations (1) to (19).

## 2. SINGLE DEFINITE INTEGRAL

In this section, a method for finding definite integral of y=f(x) in [a,b] is presented for a continuous function f(x) using Simpson's $\frac{1}{3}-$ rule. That is, to find the value of

$$\int_{x=a}^{b} f(x)dx \tag{1}$$

subdivide the interval [a, b] using step size $h = x_{n+1} - x_n$, n=0(1)N-1 into N-subintervals and rewrite [a, b] as union of $\frac{N}{2}$- intervals of each length 2h

$$[a,b] = \cup_{n=0}^{N-2}[x_n, x_{n+2}], n = 0(2), N-2 \tag{2}$$

And the corresponding distribution table is shown in table 1.

Table 1.Distribution table for y

| x | $x_0$ | $x_1$ | $x_2$ | . | . | . | $x_{N-1}$ | $x_N$ |
|---|---|---|---|---|---|---|---|---|
| y | $y_0$ | $y_1$ | $y_2$ | . | . | . | $y_{N-1}$ | $y_N$ |

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Now from (1) and (2),

$$\int_{x=a}^{b} f(x)dx = \Sigma_{n=0}^{N-2} \int_{x_n}^{x_{n+2}} f(x)dx \tag{3}$$

Any $x\varepsilon[x_n, x_{n+2}]$, $n = 0(2)N-2$ can be written in the form $x = x_n + uh$

As $x : x_n \to x_{n+2}$ the variable $u : 0 \to 2$. And so, equation (3) becomes

$$\int_{x=a}^{b} f(x)dx = h\Sigma_{n=0}^{N-2} \int_{u=0}^{2} f(x_n + uh)du \tag{4}$$

Using Newton's interpolation formulae and $y - f(x)$ in (4),

$$y(x_n + uh) = y_n + uC_1\Delta y_n + uC_2\Delta^2 y_n + ... \tag{5}$$

$$\int_{x=a}^{b} f(x)dx = h\Sigma_{n=0}^{N-2} \int_{u=0}^{2} y(x_n + uh)du \tag{6}$$

Take only two terms of (5), equation (6) reduces to

$$\int_{x=a}^{b} f(x)dx = h\Sigma_{n=0}^{N-2} \int_{u=0}^{2} [y_n + uC_1\Delta y_n + uC_2\Delta^2 y_n]du$$

$$= h\Sigma_{n=0}^{N-2} \int_{u=0}^{2} \left[ y_n + u\Delta y_n + \frac{u(u-1)}{2}\Delta^2 y_n \right] du$$

$$= h\Sigma_{n=0}^{N-2} \left[ uy_n + \frac{u^2}{2}\Delta y_n + \frac{1}{2}\left(\frac{u^3}{3} - \frac{u^2}{2}\right)\Delta^2 y_n \right]_{u=0}^{u=2}$$

$$= h\Sigma_{n=0}^{N-2} \left[ 2y_n + \frac{2^2}{2}\Delta y_n + \frac{1}{2}\left(\frac{2^3}{3} - \frac{2^2}{2}\right)\Delta^2 y_n \right]$$

$$= h\Sigma_{n=0}^{N-2} \left[ 2y_n + 2\Delta y_n + \frac{1}{3}\Delta^2 y_n \right]$$

$$= \frac{h}{3}\Sigma_{n=0}^{N-2}[y_n + 4y_{n+1} + y_{n+2}], n = 0(2)N-2 \tag{7}$$

since $\Delta y_n = [y_{n+1} - y_n]$ and

$$\Delta^2 y_n = \Delta(y_{n+1} - y_n) = [y_{n+2} - y_{n+1}] - [y_{n+1} - y_n] = [y_n - 2y_{n+1} + y_{n+2}]$$

From (7), the definite integral is

$$\int_{x=a}^{b} f(x)dx = \frac{h}{3}\Sigma_{n=0}^{N-2}[y_n + 4y_{n+1} + y_{n+2}], n = 0(2)N-2$$

$$= \frac{h}{3}[y_0 + 4(y_1 + y_3 + \ldots + y_{N-1}) + 2(y_2 + y_4 + \ldots + y_{N-2}) + y_N]$$

$$= \frac{h}{3}[y_0 + 4\Sigma_{n=1}^{N-1} y_n + 2\Sigma_{n=2}^{N-2} y_n + y_N] \tag{8}$$

**Remark** $\int_{x=a}^{b} f(x)dx = \frac{h}{3}$ [ (value of $y = f(x)$ at left end point $x = a$) + 4(leaving left end and right end points $x = a$ and $x = b$ find the sum of the values of $y = f(x)$ at all odd suffixes ) + 2(leaving left end and right end points $x = a$ and $x = b$ find the sum of the values of $y = f(x)$ at all even suffixes ) + ( value of $y = f(x)$ at right end point $x = b$ ) ].

## 3. DOUBLE DEFINITE INTEGRAL

In this section, a method for finding definite integral of $z = f(x,y)$ in the domain [a,b]X[c,d] is presented for a continuous function $f(x,y)$ using Simpson's $\frac{1}{3}-$ rule. That is, to find the value of

$$\int_{x=a}^{b}\int_{y=c}^{d} f(x,y)dxdy \tag{9}$$

subdivide the interval [a, b] using step size $h = x_{n+1} - x_n, n = 0(1)N-1$ into $N-$ subintervals and rewrite [a, b] as union of intervals of length 2h

$$[a,b] = \cup_{n=0}^{N-2}[x_n, x_{n+2}].n = 0(2)N-2 \tag{10}$$

and, subdivide the interval [c, d] using step size $k = y_{m+1} - y_m$ $m = 0(1)M-1$ into $M-$ subintervals and rewrite [c, d] as union of intervals of length 2k

$$[c,d] = \cup_{m=0}^{M-2}[y_m, y_{m+2}], m = 0(2)M-2 \tag{11}$$

From (10) and (11), it follows that

$$[a,b]X[c,d] = \cup_{n=0}^{N-2}\cup_{m=0}^{m-2}[x_n, x_{n+2}]X[y_m, y_{m+2}] \tag{12}$$

for $n = 0(2)N-2$ and $m = 0(2)M-2$ and the corresponding distribution table is shown in table 2.

Table 2.Distribution table for $f(x,y)$

| $x \setminus y$ | $y_0$ | $y_1$ | $y_2$ | . | $y_j$ | . | $y_{M-1}$ | $y_M$ |
|---|---|---|---|---|---|---|---|---|
| $x_0$ | $f(x_0,y_0)$ | $f(x_0,y_1)$ | $f(x_0,y_2)$ | | $f(x_0,y_j)$ | | $f(x_0,y_{M-1})$ | $f(x_0,y_M)$ |
| $x_1$ | $f(x_1,y_0)$ | $f(x_1,y_1)$ | $f(x_1,y_2)$ | | $f(x_1,y_j)$ | | $f(x_1,y_{M-1})$ | $f(x_1,y_M)$ |
| $x_2$ | $f(x_2,y_0)$ | $f(x_2,y_1)$ | $f(x_2,y_2)$ | | $f(x_2,y_j)$ | | $f(x_2,y_{M-1})$ | $f(x_2,y_M)$ |
| . | | | | | | | | |
| $x_i$ | $f(x_i,y_0)$ | $f(x_i,y_1)$ | $f(x_i,y_2)$ | | $f(x_i,y_j)$ | | $f(x_i,y_{M-1})$ | $f(x_i,y_M)$ |
| . | | | | | | | | |
| $x_{N-2}$ | $f(x_{N-2},y_0)$ | $f(x_{N-2},y_1)$ | $f(x_{N-2},y_2)$ | | $f(x_{N-2},y_j)$ | | $f(x_{N-2},y_{M-1})$ | $f(x_{N-2},y_M)$ |
| $x_{N-1}$ | $f(x_{N-1},y_0)$ | $f(x_{N-1},y_1)$ | $f(x_{N-1},y_2)$ | | $f(x_{N-1},y_j)$ | | $f(x_{N-1},y_{M-1})$ | $f(x_{N-1},y_M)$ |
| $x_N$ | $f(x_N,y_0)$ | $f(x_N,y_1)$ | $f(x_N,y_2)$ | | $f(x_N,y_j)$ | | $f(x_N,y_{M-1})$ | $f(x_N,y_M)$ |

Now from (9) and (12),

$$\int_{x=a}^{b}\int_{y=c}^{d} f(x,y)dxdy = \Sigma_{n=0}^{N-2}\Sigma_{m=0}^{M-2}\int_{x_n}^{x_{n+2}}\int_{y_m}^{y_{m+2}} f(x,y)dxdy \tag{13}$$

Any $(x,y)\varepsilon[x_n,x_{n+2}]X[y_m,y_{m+2}], n = 0(2)N-2,\ m = 0(2)M-2$ can be written in the form $x = x_n + uh$ $y = y_m + vk$. As $x : x_n \to x_{n+2}$ the variable $u : 0 \to 2$ and, as $y : y_m \to y_{m+2}$. the variable $v : 0 \to 2$ and so, equation (13) becomes

$$\int_{x=a}^{b}\int_{y=c}^{d} f(x,y)dxdy = hk\Sigma_{n=0}^{N-2}\Sigma_{m=0}^{M-2}\int_{u=0}^{2}\int_{v=0}^{2} f(x_n+uh, y_m+vk)dudv \tag{14}$$

Using Newton's interpolation formulae to $f(x_n+uh, y_m+vk.)$ and taking first two terms

$$\int_{x=a}^{b}\int_{y=c}^{d} f(x,y)dxdy = hk\Sigma_{n=0}^{N-2}\Sigma_{m=0}^{M-2}\int_{u=0}^{2}\left[\int_{v=0}^{2} g(y_m+vk)dv\right]du \tag{15}$$

where $f(x_n+uh, y_m+vk) = g(y_m+vk)$ by treating first variable fixed. And using first three terms of Newton's interpolation and using equation(8)

$$g(y_m+vk) = g_m + vC_1\Delta g_m + vC_2\Delta^2 g_n + \dots, \tag{16}$$

$$\int_{x=a}^{b}\int_{y=c}^{d} f(x,y)dxdy$$

$$= hk\Sigma_{n=0}^{N-2}\Sigma_{m=0}^{M-2}\int_{u=0}^{2}\left[\int_{v=0}^{2}(g_m + vC_1\Delta g_m + uC_2\Delta^2 g_m)dv\right]du \tag{17}$$

$$= \frac{hk}{3}\Sigma_{n=0}^{N-2}\int_{u=0}^{2}[g_0 + 4\Sigma_{n=1}^{M-1}g_m + 2\Sigma_{n=2}^{M-2}g_m + g_M]du$$

$$= \frac{hk}{3}\Sigma_{n=0}^{N-2}\left[\int_{u=0}^{2}g_0\,du + 4\Sigma_{m=1}^{M-2}\int_{u=0}^{2}g_m du + 2\Sigma_{m=2}^{M-2}\int_{u=0}^{2}g_m du + \int_{u=0}^{1}g_M du\right] \tag{18}$$

since, $f(x_n+uh, y_m+vk) = g(y_m+vk)$ , hiding first variable, $g_0 = f(x_n+uh, y_0) = p(x_n+uh)$, hiding second variable, $g_m = f(x_n+uh, y_m+vk) = q(x_n+uh)$, hiding second variable, $m = odd$, $g_m = f(x_n+uh, y_m+vk) = q(x_n+uh)$, hiding second variable, $m = even$ and $g_M = f(x_n+uh, y_M) = r(x_n+uh)$, hiding second variable.
Again using first three terms of Newton's interpolation to the functions p, q and r , and using (8) one have

$$\int_{x=a}^{b}\int_{y=c}^{d} f(x,y)dxdy =$$

$$\frac{hk}{9}\Bigg[\Big(f(x_0,y_0) + 4\Sigma_{n=odd}^{N-1}f(x_n,y_0) + 2\Sigma_{n=even}^{N-2}f(x_n,y_0) + f(x_N,y_0)\Big)+$$

$$\Big(f(x_0,y_M) + 4\Sigma_{n=odd}^{N-1}f(x_n,y_M) + 2\Sigma_{n=even}^{N-2}f(x_n,y_M) + f(x_N,y_M)\Big)+$$

$$\Big(4\Sigma_{m=odd}^{M-1}f(x_0,y_m) + 2\Sigma_{m=even}^{M-2}f(x_0,y_m)\Big)+$$

$$\Big(4\Sigma_{m=odd}^{M-1}f(x_N,y_m) + 2\Sigma_{m=even}^{M-2}f(x_N,y_m)\Big)+$$

$$\Big(16\Sigma_{n=odd}^{N-1}\Sigma_{m=odd}^{M-1}f(x_n,y_m) + 8\Sigma_{n=odd}^{N-1}\Sigma_{m=even}^{M-2}f(x_n,y_m)+$$

$$8\Sigma_{n=even}^{N-2}\Sigma_{m=odd}^{M-1}f(x_n,y_m) + 4\Sigma_{n=even}^{N-2}\Sigma_{m=even}^{M-2}f(x_n,y_m)\Big)\Bigg] \tag{19}$$

**Remark** Two easy ways to find the double integral (19) are

$$\int_{x=a}^{b}\int_{y=c}^{d} f(x,y)dxdy = \frac{hk}{9}\Bigg[\frac{3}{h}\lfloor\, apply\ Simpson\ rule\ in\ the\ first\ row\rfloor+$$

$$\frac{3}{h}\lfloor\, apply\ Simpson\ rule\ on\ the\ last\ row\rfloor+$$

$$(4\lfloor sum\ of\ all\ entrices\ with\ odd\ suffixes\ in\ first\ column\ leaving\ first\ and\ last\ entrices\rfloor+$$

$$2\lfloor sum\ of\ all\ entrices\ with\ even\ suffixes\ in\ first\ column\ leaving\ first\ and\ last\ entrices\rfloor)+$$

$$(4\lfloor sum\ of\ all\ entrices\ with\ odd\ suffixes\ in\ last\ column\ leaving\ first\ and\ last\ entrices\rfloor+$$

$$2\lfloor sum\ of\ all\ entrices\ with\ even\ suffixes\ in\ last\ column\ leaving\ first\ and\ last\ entrices\rfloor)+$$

$$leaving\ first\ and\ last\ columns\ and\ leaving\ first\ and\ last\ rows\ find$$

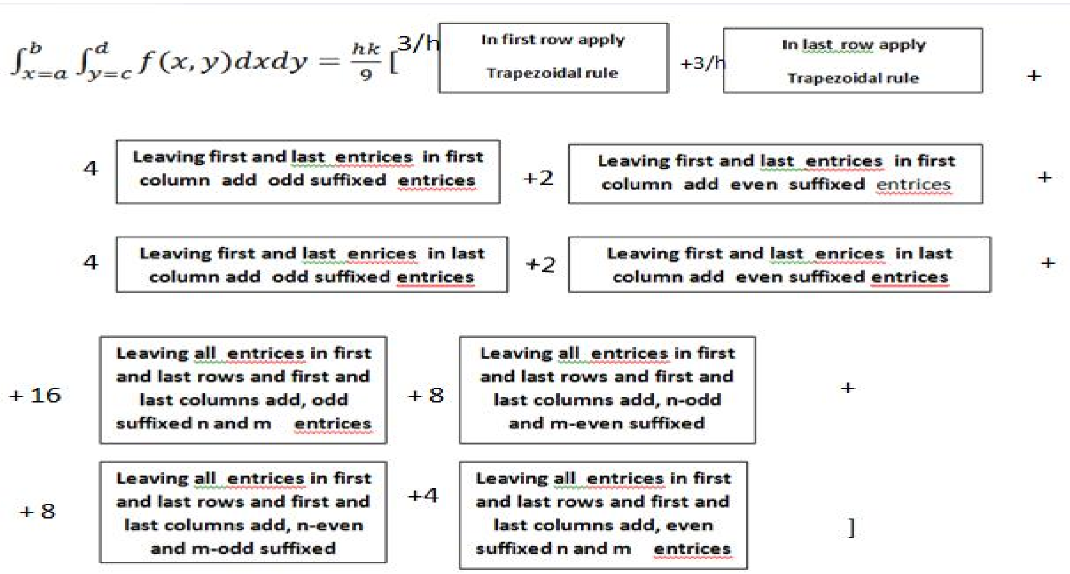
$$\{(16[sum\ of\ all\ entrices\ with\ odd\ suffixes\ of\ n\ and\ m]+$$
$$8[sum\ of\ all\ entrices\ with\ odd\ suffixes\ of\ n\ and\ even\ suffixes\ of\ m]+$$
$$8[sum\ of\ all\ entrices\ with\ even\ suffixes\ of\ n\ and\ odd\ suffixes\ of\ m]+$$
$$\left.4[sum\ of\ all\ entrices\ with\ even\ suffixes\ of\ n\ and\ m])\}\right]$$

and

$$\int_{x=a}^{b}\int_{y=c}^{d} f(x,y)dxdy = \frac{hk}{9}\left[\frac{3}{h}\lfloor apply\ Simpson\ rule\ in\ the\ first\ column\rfloor+\right.$$
$$\frac{3}{h}\lfloor apply\ Simpson\ rule\ on\ the\ last\ column\rfloor+$$
$$(4\lfloor sum\ of\ all\ entrices\ with\ odd\ suffixes\ in\ first\ row\ leaving\ first\ and\ last\ entrices\rfloor+$$
$$2\lfloor sum\ of\ all\ entrices\ with\ even\ suffixes\ in\ first\ row\ leaving\ first\ and\ last\ entrices\rfloor)+$$
$$(4\lfloor sum\ of\ all\ entrices\ with\ odd\ suffixes\ in\ last\ row\ leaving\ first\ and\ last\ entrices\rfloor+$$
$$2\lfloor sum\ of\ all\ entrices\ with\ even\ suffixes\ in\ last\ row\ leaving\ first\ and\ last\ entrices\rfloor)+$$
$$leaving\ first\ and\ last\ columns\ and\ leaving\ first\ and\ last\ rows\ find$$
$$\{(16[sum\ of\ all\ entrices\ with\ odd\ suffixes\ of\ n\ and\ m]+$$
$$8[sum\ of\ all\ entrices\ with\ odd\ suffixes\ of\ n\ and\ even\ suffixes\ of\ m]+$$
$$8[sum\ of\ all\ entrices\ with\ even\ suffixes\ of\ n\ and\ odd\ suffixes\ of\ m]+$$
$$\left.4[sum\ of\ all\ entrices\ with\ even\ suffixes\ of\ n\ and\ m])\}\right]$$

## 4. AN ALGORITHM TO FIND THE DEFINITE DOUBLE INTEGRAL

This section presents an algorithm to evaluate the definite double integral using Simpson's $\frac{1}{3}-$ rule in two different pictorial forms instead of writing in steps. The pictorial forms of algorithm are



and

$$\int_{x=a}^{b}\int_{y=c}^{d} f(x,y)\,dx\,dy = \frac{hk}{9}\Big[3/k\ \boxed{\begin{array}{c}\text{In first column apply}\\ \text{Trapezoidal rule}\end{array}} + 3/k\ \boxed{\begin{array}{c}\text{In last column apply}\\ \text{Trapezoidal rule}\end{array}} +$$

$$+4\ \boxed{\begin{array}{c}\text{Leaving first and last entrices in first}\\ \text{row add odd suffixed entrices}\end{array}} + 2\ \boxed{\begin{array}{c}\text{Leaving first and last entrices in first}\\ \text{row add even suffixed entrices}\end{array}} +$$

$$+4\ \boxed{\begin{array}{c}\text{Leaving first and last enrices in last}\\ \text{row add odd suffixed entrices}\end{array}} + 2\ \boxed{\begin{array}{c}\text{Leaving first and last enrices in last row}\\ \text{add even suffixed entrices}\end{array}} +$$

$$+16\ \boxed{\begin{array}{c}\text{Leaving all entrices in first}\\ \text{and last rows and first and}\\ \text{last columns add, odd}\\ \text{suffixed n and m entrices}\end{array}} + 8\ \boxed{\begin{array}{c}\text{Leaving all entrices in first}\\ \text{and last rows and first and}\\ \text{last columns add, n-odd}\\ \text{and m-even suffixed}\end{array}} +$$

$$+8\ \boxed{\begin{array}{c}\text{Leaving all entrices in first}\\ \text{and last rows and first and}\\ \text{last columns add, n-even}\\ \text{and m-odd suffixed}\end{array}} + 4\ \boxed{\begin{array}{c}\text{Leaving all entrices in first}\\ \text{and last rows and first and}\\ \text{last columns add, even}\\ \text{suffixed n and m entrices}\end{array}}\ \Big]$$

Using these pictorial forms, in next section, some single and double integrals are evaluated. Above two pictorial forms will generate the same value.

## 5. NUMERICAL EXPERIMENT

In this section, five single and four double integrals using Simpson's $\frac{1}{3}-$ - rule are evaluated using remarks in section 2 and section 3 and pictorial form of algorithm in section 4. The step size used in all problems is $h = \frac{1}{4}$ and $k = \frac{1}{4}$.

**Problem 1**

Evaluate $\int_{x=0}^{1} e^{-x^2}dx$

The corresponding distribution table is

| x | 0 | 0.25 | 0.5 | 0.75 | 1.0 |
|---|---|---|---|---|---|
| y | 1 | 0.9394 | 0.7788 | 0.5697 | 0.3678 |

Using remarks in section 2,

$$\int_{x=0}^{1} e^{-x^2}dx = \frac{0.25}{3}[1+4(0.9394+0.5697)+2(0.7788)+0.3678] = 0.61638 \text{ units.}$$

**Problem 2**

Evaluate $\int_{x=0}^{1} e^{-x}dx$

The corresponding distribution table is

| x | 0 | 0.25 | 0.5 | 0.75 | 1.0 |
|---|---|---|---|---|---|
| y | 1 | 0.778 | 0.606 | 0.472 | 0368 |

Using remarks in section 2,

$$\int_{x=0}^{1} e^{-x}dx = \frac{0.25}{3}[1+4(0.778+0.472)+2(0.606)+0.368] = 0.6316\text{units}.$$

**Problem 3**

Evaluate $\int_{x=0}^{1} x^2 dx$

The corresponding distribution table is

| x | 0 | 0.25 | 0.5 | 0.75 | 1.0 |
|---|---|---|---|---|---|
| y | 0 | 0.0635 | 0.25 | 0.5625 | 1 |

Using remarks in section 2,

$$\int_{x=0}^{1} x^2 dx = \frac{0.25}{3}[0+4(0.0655+0.5625)+2(0.25)+1] = 0.33433 \text{ units}.$$

**Problem 4**

Evaluate $\int_{x=0}^{1} sin(x)dx$

The corresponding distribution table is

| x | 0 | 0.25 | 0.5 | 0.75 | 1.0 |
|---|---|---|---|---|---|
| y | 0 | 0.2474 | 0.4794 | 0.6816 | 0.8414 |

Using remarks in section 2,

$$\int_{x=0}^{1} sin(x)dx = \frac{0.25}{3}[0+4(0.2474+0.6816)+2(0.4794)+0.8414] = 0.4596 \text{ units}.$$

**Problem 5**

Evaluate $\int_{x=0}^{1} cos(x)dx$

The corresponding distribution table is

| x | 0 | 0.25 | 0.5 | 0.75 | 1.0 |
|---|---|---|---|---|---|
| y | 1 | 0.9689 | 0.8775 | 0.7316 | 0.5403 |

Using remarks in section 2,

$$\int_{x=0}^{1} cos(x)dx = \frac{0.25}{3}[1+4(0.9689+0.7316)+2(0.8775)+0.5403] = 0.84144 \text{ units}.$$

**Problem 6**

Evaluate $\int_{x=0}^{1}\int_{y=0}^{1} e^{-x^2}e^{-y^2}dxdy$

The corresponding distribution table is

| x/y | 0 | 0.25 | 0.5 | 0.75 | 1.0 |
|---|---|---|---|---|---|
| 0 | 1 | 0.9394 | 0.7788 | 0.5697 | 0.3678 |
| 0.25 | 0.9394 | 0.8824 | 0.7316 | 0.5351 | 0.3455 |
| 0.5 | 0.7788 | 0.7316 | 0.6065 | 0.4436 | 0.2864 |
| 0.75 | 0.5697 | 0.5351 | 0.4436 | 0.3246 | 0.2095 |
| 1.0 | 0.3678 | 0.3455 | 0.2864 | 0.2095 | 0.1353 |

Using the pictorial form of algorithm

$$\int_{x=0}^{1}\int_{y=0}^{1} e^{-x^2}e^{-y^2}dxdy =$$

$$\frac{0.25x0.25}{9}[7.3965+3.2989+6.8152+2.7928+36.4832+9.4016+9.4016+2.426] = 0.55477\text{sq. units.}$$

**Problem 7**

Evaluate $\int_{x=0}^{1}\int_{y=0}^{1} e^{-x}e^{-y}dxdy$
The corresponding distribution table is

| x/y | 0 | 0.25 | 0.5 | 0.75 | 1.0 |
|---|---|---|---|---|---|
| 0 | 1 | 0.778 | 0.606 | 0.472 | 0.368 |
| 0.25 | 0.778 | 0.606 | 0.472 | 0.368 | 0.286 |
| 0.5 | 0.606 | 0.472 | 0.368 | 0.286 | 0.223 |
| 0.75 | 0.472 | 0.368 | 0.286 | 0.223 | 0.173 |
| 1.0 | 0.368 | 0.286 | 0.223 | 0.173 | 0.125 |

Using the pictorial form of algorithm

$$\int_{x=0}^{1}\int_{y=0}^{1} e^{-x}e^{-y}dxdy =$$

$$\frac{0.25x0.25}{9}[7.58+2.775+6.212+2.282+25.072++6.064+6.064+1.472] = 0.3994 \text{ sq. units.}$$

**Problem 8**

Evaluate $\int_{x=0}^{1}\int_{y=0}^{1} x^2y^2dxdy$
The corresponding distribution table is

| x/y | 0 | 0.25 | 0.5 | 0.75 | 1.0 |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0.25 | 0 | 0.0039 | 0.0156 | 0.0351 | 0.0625 |
| 0.5 | 0 | 0.0156 | 0.0625 | 0.1406 | 0.25 |
| 0.75 | 0 | 0.0351 | 0.1406 | 0.3164 | 0.5625 |
| 1.0 | 0 | 0.0625 | 0.25 | 0.5625 | 1 |

Using the pictorial form of algorithm

$$\int_{x=0}^{1}\int_{y=0}^{1} x^2y^2dxdy =$$

$$\frac{0.25x0.25}{9}[0+3+0+3+6.248+1.2496+1.2496+0.25] = 0.1042 \text{ sq. units.}$$

**Problem 9**

Evaluate $\int_{x=0}^{1}\int_{y=0}^{1} sin(x)cos(y)dxdy$

The corresponding distribution table is

| x/y | 0 | 0.25O | 0.5E | 0.75O | 1.0 |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0.25O | 0.2174 | 0.2397 | 0.2170 | 0.1809 | 0.1338O |
| 0.5E | 0.4794 | 0.4644 | 0.4206 | 0.3504 | 0.2590E |
| 0.75O | 0.6816 | 0.6604 | 0.5981 | 0.4986 | 0.3682O |
| 1.0 | 0.8414 | 0.8132 | 0.7383 | 0.6155 | 0.4546 |

Using the pictorial form of algorithm

$$\int_{x=0}^{1}\int_{y=0}^{1} sin(x)cos(y)dxdy =$$

$$\frac{0.25x0.25}{9}[5.3962+2.9805+0+7.252+25.2736+6.5208+6.5184+1.6824] = 0.3862 \text{ sq. units.}$$

It is observed that the numerical solution using traditional Simpson's $\frac{1}{3}-$ rule for ordinary differential equation do not match with the definite integral of a single integral of a function f(x) of section 2. This motivates to design a new Simpson's $\frac{1}{3}-$ rule for the numerical solution of ordinary differential equations using the proof used in this paper.

## 6. CONCLUSION

In this paper, an algorithm for Simpson's $\frac{1}{3}-$ rule is presented, theoretically proved, pictorially described and easy to implement. The approach applied to single integral is extended to double integral. This approach can be extended to higher dimensions. Simpson's $\frac{1}{3}-$ rule derivation of this paper gives an opening to find another Simpson's $\frac{1}{3}-$ rule which is different from the traditional Simpson's $\frac{1}{3}-$ rule to solve stiff and non-stiff ordinary and partial differential equations. Research is going on in this direction by the author.

## REFERENCES


[1] A.Ralston and P.Rabinowitz, First Course in Numerical Analysis, McGraw-Hill Book Company, 1986.

[2] Dhananjaya Reddy, Cascade and System Reliability for Exponential Distributions, DJ Journal of Engineering and Applied Mathematics, Vol. 2, No. 2, 2016, pp. 1-8, https://dx.doi.org/10.18831/djmaths.org/2016021001

[3] N.S.Bakhvalov, Numerical Methods, Mir Publishing Company, Moscow, 1977.

[4] E.Ahmed, M.Safan and H.Nabih, On Evolutionary Games with Periodic Payoffs, DJ Journal of Engineering and Applied Mathematics, Vol. 1, No. 1, 2015, pp. 1-3, https://dx.doi.org/10.18831/djmaths.org/2015011001