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RESEARCH ARTICLE

Some Results on Integer Edge Cordial Graph

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ABSTRACT

An integer edge cordial labeling of a graph G with edge set E is an injective map f from E to $[\frac{-q}{2}..\frac{q}{2}]^*$ or $[-\lfloor\frac{q}{2}\rfloor..\lfloor\frac{q}{2}\rfloor]$ as q is even or odd, which induces a vertex labeling $f^*: V \to \{0,1\}$ such that, a vertex u is assigned the label 1 if $\sum_i f^*(e_i) \geq 0$, and 0 otherwise and the number of vertices labeled with 1 and the number of vertices labeled with 0 differs atmost by 1. If a graph has integer edge cordial labeling then it is called integer edge cordial graph. In this paper, we introduce the concept of integer edge cordial labeling and prove that some standard graphs such as path, cycle, wheel, helm, closed helm, star graph $K_{1,n}$ are integer edge cordial and $K_{n,n}$ is not integer edge cordial. It is also proved that $K_{nn}\backslash M$ is integer edge cordial if n is even where M is a perfect matching of K_{nn}.

Keywords: Cordial labeling, Integer edge cordial labeling, Cordial graph, Vertex labeling, Star graph.

1. INTRODUCTION

By a graph we mean a finite undirected graph without loops and multiple edges. For terms not defined here we refer to Harary [7].

Cordial graph was first introduced by I.Cahit [1,2] in 1987 as a weaker version of graceful and harmonious graphs and was based on $\{0,1\}$ binary labeling of vertices. He showed that (i) every tree is cordial (ii) K_n is cordial if and only if $n \leq 3$ (iii) $K_{r,s}$ is cordial for all r and s (iv) the wheel W_n is cordial if and only if $n \equiv 3$ (mod 4) (v) C_n is cordial if and only if $n \neq 2$ (mod 4) (vi) an Eulerian graph is not cordial if its size is congruent to 2 modulo 4. Other types of cordial graphs were considered in [3,8-13]. For more related results on cordial graphs one can refer to Gallian [4-6].

Definition 1.1 [1]

Consider a labeling f of G where $N = \{0,1\}$, and the induced edge-labeling $\overline{f}$ is given by $\overline{f}(\{u,v\}) = |f(u) - f(v)|, \overline{N} = \{0,1\}$. We call such a labeling cordial if the following condition is satisfied:

$|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where v_f (i) and e_f (i), $i = 0$, 1 is the number of vertices and edges of G respectively with label i (under f and $\overline{f}$ respectively).

A graph G is called cordial if it admits cordial labeling.

Definition 1.2 [14]

Let f be a binary edge labeling of graph $G = \{V,E\}$, i.e. $f : E(G) \to \{0,1\}$, and the induced labeling is given as $f(v) = \sum_{\forall u} f(u,v)(mod2)$ where $v\varepsilon V$ and $\{u,v\}\varepsilon E$. f is called an E-cordial labelling of G if the following conditions are satisfied:

1) $|e_f(0) - e_f(1)| \leq 1$

2) $|v_f(0) - v_f(1)| \leq 1$

where $e_f(0), e_f(1)$ denote the number of edges, and $v_f(0), v_f(1)$ denote the number of vertices with $0's$ and $1's$ respectively. The graph G is called E-cordial if it admits E-cordial labeling.

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In 1997 Yilmaz and Cahit [14] have introduced E-cordial labeling as a weaker version of edge–graceful labeling. They proved that the trees with n vertices, K_n,C_n are E-cordial if and only if $n \not\equiv 2$(mod 4) while $K_{m,n}$ admits E-cordial labeling if and only if $m+n \not\equiv 2$(mod4).

We introduce the concept of integer edge cordial labeling and we prove that some standard graphs such as cycle C_n, some standard graphs such as path P_n, cycle C_n, wheel W_n, n > 3, helm H_n, closed helm CH_n, star graph $K_{1,n}$ are edge integer cordial and $K_{n,n}$ is not integer cordial. It is also proved that $K_{nn} \backslash M$ is integer cordial if n is even where M is a perfect matching of K_{nn}.

Definition 1.3

The Wheel W_n is the join of the graphs C_n and K_1.i.e, $W_n = C_n + K_1$. Here the vertices corresponding to C_n are called rim vertices and C_n is called rim of W_n while the vertex corresponding to K_1 is called apex vertex.

Definition 1.4

A helm H_n, $n \geq 3$ is the graph obtained from the wheel W_n by adding a pendant edge at each rim vertex.

Definition 1.5

The flower Fl_n is the graph obtained from a helm H_n by joining each pendant vertex to apex vertex of the helm. Here we consider rim vertices as internal vertices.

Definition 1.6

The closed helm CH_n is the graph obtained from a helm H_n by joining each pendant vertex to form a cycle.

Notation 1.7

(i) $[-x..x]$=$\{t/t$ is an integer and $|t| \leq x\}$

(ii) $[-x..x]^* = [-x..x] - \{0\}$

Definition 1.8

Let G=(V,E) be a simple connected graph with p vertices and q edges. Let $f : E \rightarrow [\frac{-q}{2}..\frac{q}{2}]^*$ or $[-\lfloor\frac{q}{2}\rfloor..\lfloor\frac{q}{2}\rfloor]$ as q is even or odd, be an injective map which induces a vertex labeling f^* such that

$$f^*(v_i) = \begin{cases} 0 & if \quad \sum_i f(e_i) \leq 0 \\ 1 & if \quad \sum_i f(e_i) > 0 \end{cases}$$

Let $v_f^*(i)$ = number of vertices labeled with i, where i = 0 or 1. f^* is said to be integer edge cordial if $|v_f^*(0) - v_f^*(1)| \leq 1$. A graph is said to be integer edge cordial graph if it admits integer edge cordial labeling. Figure 1 shows the integer edge cordial graph.

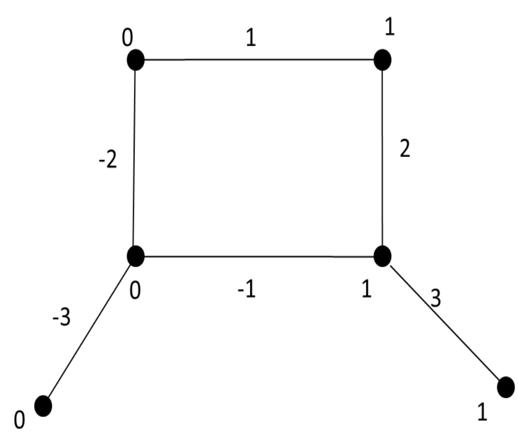

Figure 1.Integer edge cordial graph

2. MAIN RESULTS

Theorem 2.1

Every path $P_n, n \geq 3$ is integer edge cordial.

Proof

Let $V = \{v_1, v_2, v_3, ..., v_n\}$ be the vertex set and $E = \{e_1, e_2, e_3, ..., e_{n-1}\}$ be the edge set, where $e_i = v_i v_{i+1}; 1 \leq i \leq n-1$. Here $p = n$ and $q = n-1$. We consider two cases:

Case(i) q is odd (n even)

We define $f : E \to [-\lfloor q/2 \rfloor .. \lfloor q/2 \rfloor]$

$f(v_i v_{i+1}) = -i; 1 \leq i \leq \frac{n}{2} - 1$

$f(v_{\frac{n}{2}} v_{\frac{n}{2}+1}) = 0$

$f(v_{\frac{n}{2}+i} v_{\frac{n}{2}+1+i}) = i; 1 \leq i \leq \frac{n}{2} - 2$

Let us consider the vertex $v_{\frac{n}{2}+1}$.

Here the edge $f(v_{\frac{n}{2}} v_{\frac{n}{2}+1})$ is given label 0 and $f(v_{\frac{n}{2}+1} v_{\frac{n}{2}+2})$ is labeled as 1. Then $f(v_{\frac{n}{2}+1}) = 0 + 1 = 1$, which is a positive integer.

Since the edge $f(v_{\frac{n}{2}} v_{\frac{n}{2}+1})$ is labeled with 0, the vertex $v_{\frac{n}{2}}$ receives label 0, and $v_{\frac{n}{2}+2}$ receives label 1.

Here $\frac{p}{2} = \frac{n}{2}$ vertices receives label 0. Also $\frac{p}{2} = \frac{n}{2}$ vertices receive label 1.

That is, $v_f^*(0) = v_f^*(1) = \frac{p}{2}$.

Hence $|v_f^*(0) - v_f^*(1)| = 0$.

Case (ii) q is even (n odd)

We define $f : E \to [-\lfloor \frac{q}{2} \rfloor .. \lfloor \frac{q}{2} \rfloor]^*$.

Consider the following labeling:

$f(v_i v_{i+1}) = -i \; ; \; 1 \leq i \leq \lfloor \frac{n}{2} \rfloor$

$f(v_{\lfloor \frac{n}{2} \rfloor + i} v_{\lceil \frac{n}{2} \rceil + i}) = i \; ; \; 1 \leq i \leq \lfloor \frac{n}{2} \rfloor$

Let us consider the edge $(v_{\lfloor \frac{n}{2} \rfloor} v_{\lceil \frac{n}{2} \rceil})$ which is labeled as $(-\lfloor \frac{n}{2} \rfloor)$and $(v_{\lceil \frac{n}{2} \rceil} v_{\lceil \frac{n}{2} \rceil + 1})$ is labeled as 1.

Hence, $f(v_{\lceil \frac{n}{2} \rceil + 1}) = \lfloor \frac{n}{2} \rfloor + 1 = \frac{-n+3}{2}$.

When $n > 3, f(v_{\lceil \frac{n}{2} \rceil + 1}))$ receives label 0. Therefore, $\lceil \frac{p}{2} \rceil$ vertices receives label 0 and $\lfloor \frac{p}{2} \rfloor)$ vertices receives label 1.

That is, $v_f^*(0) = \lceil \frac{p}{2} \rceil$ and $v_f^*(1) = \lfloor \frac{p}{2} \rfloor$

Hence $|v_f^*(0) - v_f^*(1)| = 1$

Case (iii) n=3

We consider the following labels, $f(v_1v_2) = -1$; $(fv_2v_3) = 1$. Therefore, the vertices receive the labels, $f(v_1) = 0$;$f(v_2) = 1$; $f(v_3) = 1$.

Here $v_f^*(0) = 1$ and $v_f^*(1) = 2$

Hence $|v_f^*(0) - v_f^*(1)| = 1$

From both the cases we observe that $|v_f^*(0) - v_f^*(1)| \leq 1$

Hence path P_n is an integer edge cordial graph as shown in figure 2.

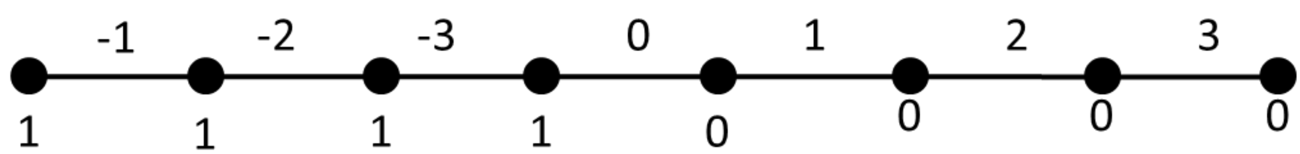

Figure 2.P_8 is integer edge cordial

Theorem 2.2

Every cycle $C_n(n \geq 3)$ is an integer edge cordial.

Proof

Let $V = \{v_1, v_2, v_3, ..., v_n\}$ and $E = \{e_1, e_2, e_3, ..., e_n\}$ be the vertex set $e_i = v_iv_{i+1}$; $1 \leq i \leq n$. Here $p = n$ and $q = n$. We consider three cases:

Case (i) n = 3

Let us consider the labels, $f(v_1v_2) = -1$; $f(v_2v_3) = 1$; $f(v_3v_1) = 0$ and so receive vertex labels as, $f(v_1) = 0; f(v_2) = 1; f(v_3) = 1$.

By considering these edge labeling we observe that the vertices v_2 and v_3 receive positive integers and v_1 receive negative integer.

That is, $v_f^*(0) = 1$ and $v_f^*(1) = 2$.

Hence, $|v_f^*(0) - v_f^*(1)| = 1$.

Case (ii) q is even (n even)

We define $f : E \to [-\lfloor \frac{q}{2} \rfloor .. \lfloor \frac{q}{2} \rfloor]^*$

$f(v_iv_{i+1}) = -i$; $1 \leq i \leq \frac{n}{2}$

$f(v_{\frac{n}{2}}v_{\frac{n}{2}+1}) = \frac{n}{2}$

$f(v_{\frac{n}{2}+i}v_{\frac{n}{2}+1+i}) = i; 1 \leq i \leq \frac{n}{2} - 1$

Here, if $f(v_1v_2) = -1$ and $f(v_1v_n) = \frac{n}{2}$, then the vertex $f(v_1) = f(v_1v_2) + f(v_1v_n) = -1 + \frac{n}{2} = \frac{n-2}{2}$.

When $n \geq 3, f(v_1)$ receives positive label.

Let us consider. Here the edge $f(v_{\frac{n}{2}}v_{\frac{n}{2}+1}) = -\frac{n}{2}$ and $(v_{\frac{n}{2}+1}v_{\frac{n}{2}+2}) = 1$. Then $(v_{\frac{n}{2}+1}) = -\frac{n}{2} + 1 = \frac{-n+2}{2}$, which is a negative integer when $n \geq 3$. Hence we label it as 0.

Thus $\frac{p}{2} = \frac{n}{2}$ vertices receives label 0 and also $\frac{p}{2} = \frac{n}{2}$ vertices receives label 1

That is, $v_f^*(0) = v_f^*(1) = \frac{p}{2}$

Hence $|v_f^*(0) - v_f^*(1)| = 0$

Case (iii) q is odd (n odd)

We define $f : E \to [-\lfloor \frac{q}{2} \rfloor .. \lfloor \frac{q}{2} \rfloor]$.

The labeling is as follows:

$f(v_iv_{i+1}) = -i; 1 \leq i \leq \lfloor \frac{n}{2} \rfloor$

$f(v_{\lfloor \frac{n}{2} \rfloor + i}v_{\lceil \frac{n}{2} \rceil + i}) = i; 1 \leq i \leq \lfloor \frac{n}{2} \rfloor$

$f(v_nv_1) = 0.$

From the above labeling we observe that $\lceil \frac{n}{2} \rceil = \lceil \frac{p}{2} \rceil$ receives label 0 and $\lfloor \frac{n}{2} \rfloor = \lfloor \frac{p}{2} \rfloor$ receives label 1. Thus $v_f^*(0) = \lceil \frac{p}{2} \rceil$ and $v_f^*(1) = 1$.

Hence $|v_f^*(0) - v_f^*(1)| = 1$

From all the cases we observe that that $|v_f^*(0) - v_f^*(1) \leq 1$.

Hence cycle C_n is an integer edge cordial graph as shown in figure 3.

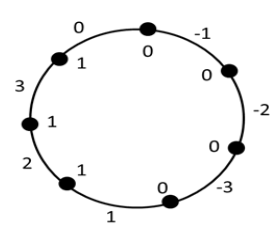

Figure 3.C_7 is edge integer cordial cordial graph

Theorem 2.3

$K_{1,n}$ is integer edge cordial if n is even.

Proof

Let $K_{1,n}$ be the star graph and $G = K_{1,n}$. Let u be the apex vertex and $\{v_1, v_2, v_3, ..., v_n\}$ be the pendant vertices. Here $p = n+1$ and $q = n$. We define $f : E \to [-\lfloor \frac{q}{2} \rfloor .. \lfloor \frac{q}{2} \rfloor]^*$

We define $f(uv_i) = -i$; $1 \leq i \leq \frac{n}{2}$

$f(uv_{\frac{n}{2}+i}) = i$; $1 \leq i \leq \frac{n}{2}$

Since $f^*(v) = \sum_{i=1}^{n} f(uv_i)$ the vertex v receives label 1 and other $\frac{n}{2}$ vertices receives label 1 and $\frac{n}{2}$ vertices receive label 0.

That is, $v_f^*(0) = \frac{n}{2}$ and $v_f^*(1) = \frac{n}{2} + 1$

Hence $|v_f^*(0) - v_f^*(1)| = 1$

Thus $K_{1,n}$ is integer edge cordial if n is even as shown in figure 4.

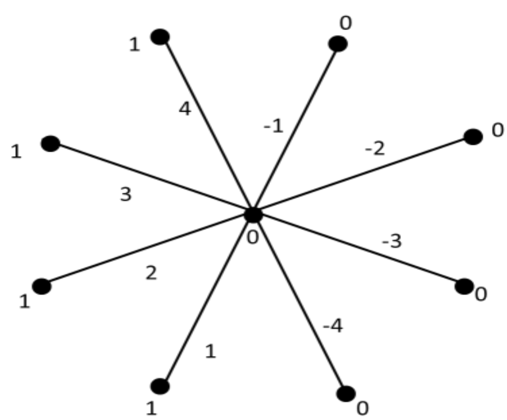

Figure 4.$K_{1,8}$ is integer edge cordial

Theorem 2.4

The wheel graph W_n, $n > 3$ is integer edge cordial

Proof

Let v be the apex vertex and $\{v_1, v_2, v_3, ..., v_n\}$ be the rim vertices. Here $p = n+1$ and $q = 2n$.

To define $f : E \rightarrow [-\lfloor \frac{q}{2} \rfloor .. \lfloor \frac{q}{2} \rfloor]^*$ we consider two cases.

Case (i) q is even(n odd)

$f(v_i v_{i+1}) = -i; 1 \leq i \leq \lceil \frac{n}{2} \rceil$

$f(v_{\lceil \frac{n}{2} \rceil + i} v_{\lceil \frac{n}{2} \rceil + 1 + i}) = \lceil \frac{n}{2} \rceil + i; 1 \leq i \leq \lfloor \frac{n}{2} \rfloor - 1$

$f(v_n v_1) = n$

$f(vv_i) = f(v_{\lceil \frac{n}{2} \rceil} v_{\lceil \frac{n}{2} \rceil + 1}) - i$; $1 \leq i \leq \lfloor \frac{n}{2} \rfloor$

$f(vv_{\lfloor \frac{n}{2} \rfloor + i}) = \lceil \frac{n}{2} \rceil + 1 - i$; $\lceil \frac{n}{2} \rceil \leq i \leq 1$

From the above we observe that $\frac{p}{2}$ vertices receive label 0 and $\frac{p}{2}$ vertices receive label 1.

That is, $v_f^*(0) = v_f^*(1) = \frac{p}{2}$

Hence $|v_f^*(0) - v_f^*(1)| = 0$

Case (ii) q is even (n even)

$f(v_i v_{i+1}) = -i; 1 \leq i \leq \frac{n}{2}$

$f(v_{\frac{n}{2}+i} v_{\frac{n}{2}+1+i}) = \frac{n}{2} + i; 1 \leq i \leq \frac{n}{2}$

$f(v_n v_1) = \frac{n}{2} + 1 - i; 1 \leq i \leq \frac{n}{2}$

$f(vv_i) = f(v_{\frac{n}{2}} v_{\frac{n}{2}+1}) - i; 1 \leq i \leq \frac{n}{2}$

$f(vv_{\frac{n}{2}+i}) = \frac{n}{2} + i; 1 \leq i \leq \frac{n}{2}$

Here $\lceil \frac{p}{2} \rceil$ vertices receives label 1 and $\lfloor \frac{p}{2} \rfloor$ vertices receives label 0. Thus, $v_f^*(1) = \lceil \frac{p}{2} \rceil$ and $v_f^*(0) = \lfloor \frac{p}{2} \rfloor$

Hence $|v_f^*(0) - v_f^*(1)| = 1$

From both the cases we observe that $|v_f^*(0) - v_f^*(1)| \leq 1$

Thus W_n is integer edge cordial as shown in figure 5.

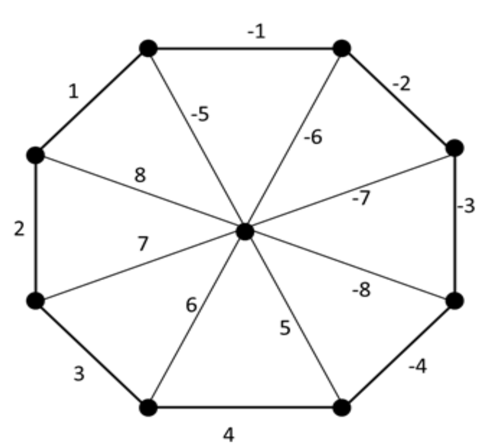

Figure 5.W_8 is integer edge cordial

Theorem 2.5

Closed Helm graph CH_n is integer edge cordial.

Proof

Let v be the apex vertex and $v_1, v_2, v_3, ..., v_n$ be the vertices of inner cycle and $u_1, u_2, u_3, ..., u_n$ be the vertices of outer cycle of CH_n. Here $p = 2n + 1$ and $q = 4n$.

To define $f : E \to [-\lfloor \frac{q}{2} \rfloor .. \lfloor \frac{q}{2} \rfloor]^*$ we consider

$f(vv_i) = -i; 1 \leq i \leq n$

$f(v_i v_{i+1}) = f(vv_n) - i; 1 \leq i \leq n$

$f(v_i u_i) = i; 1 \leq i \leq n$

$f(u_i u_{i+1}) = f(v_n u_n) + i; 1 \leq i \leq n$

Let us consider the apex vertex v.

Now,

$f(v) = \sum_{i=1}^{n} f(vv_i) = \sum_{i=1}^{n} (-i) < 0.$

$f(u_i) = \sum_{i=1}^{n} [f(v_i u_i) + f(u_i u_{i+1})] > 0$

$f(v_i) = \sum_{i=1}^{n} [f(vv_i) + f(u_i v_i) + f(v_i v_{i+1})] < 0$

Hence we observe that $v_f^*(1) = \lfloor \frac{p}{2} \rfloor$ and $v_f^*(0) = \lceil \frac{p}{2} \rceil$

Hence $|v_f^*(0) - v_f^*(1)| = 1$

Therefore CH_n is integer edge cordial as shown in figure 6.

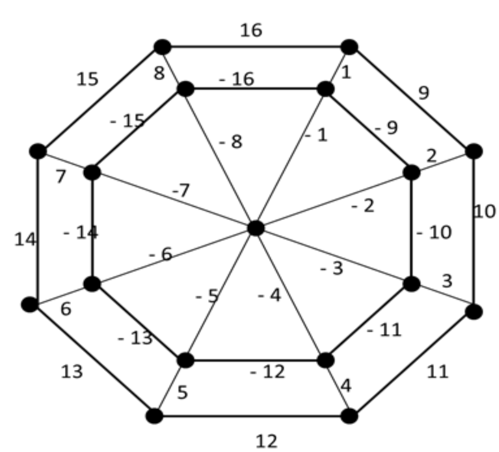

Figure 6.CH_8 is integer edge cordial

Theorem 2.6

Helm graph H_n is integer edge cordial.

Proof

Let v be the apex vertex, $\{v_1, v_2, v_3, ..., v_n\}$ be the vertices of inner cycle; $\{u_1, u_2, u_3, ..., u_n\}$ be the rim vertices. Let $H_n = G$ then $p = 2n + 1$ and $q = 3n$. We consider two cases:

Case (i) q is even (n even)

We define $f : E \to [-\lfloor \frac{q}{2} \rfloor .. \lfloor \frac{q}{2} \rfloor]^*$

$f(vv_i) = -i; 1 \leq i \leq n$

$f(v_i v_{i+1}) = -i; 1 \leq i \leq n-1$

$f(v_1 v_n) = n$

$f(v_i u_i) = f(vv_n) - i; 1 \leq i \leq \frac{n}{2}$

$f(v_{\frac{n}{2}+i} u_{\frac{n}{2}+i}) = f(vv_n) + i; 1 \leq i \leq \frac{n}{2}$

It is seen that $\lceil \frac{p}{2} \rceil$ vertices receives label 1 and $\lfloor \frac{p}{2} \rfloor$ vertices receives label 0. That is, $v_f^*(1) = \lceil \frac{p}{2} \rceil$ and $v_f^*(0) = \lfloor \frac{p}{2} \rfloor$

Hence $|v_f^*(0) - v_f^*(1)| = 1$

Case (ii) q is odd (n odd)

We define $f : E \to [-\lfloor \frac{q}{2} \rfloor .. \lfloor \frac{q}{2} \rfloor]$

$f(vv_i) = -i; 1 \leq i \leq n$

$f(v_i v_{i+1}) = i; 1 \leq i \leq n-1$

$f(v_1 v_n) = n$

$f(v_1, u_i) = f(vv_n) - i; 1 \leq i \leq \lfloor \frac{n}{2} \rfloor$

$f(v_{\lfloor \frac{n}{2} \rfloor + i} u_{\lfloor \frac{n}{2} \rfloor + i}) = f(vv_n) + i; 1 \leq i \leq \lfloor \frac{n}{2} \rfloor$

$f(v_n u_n) = 0$

We observe that $\lfloor \frac{p}{2} \rfloor$ vertices receive label 0 and $\lceil \frac{p}{2} \rceil$ vertices receive label 1. That is, $v_f^*(1) = \lceil \frac{p}{2} \rceil$ and $v_f^*(0) = \lfloor \frac{p}{2} \rfloor$

Hence $|v_f^*(0) - v_f^*(1)| = 1$

From both the cases we get, $|v_f^*(0) - v_f^*(1)| = 1$

Therefore, H_n is integer edge cordial as shown in figure 7.

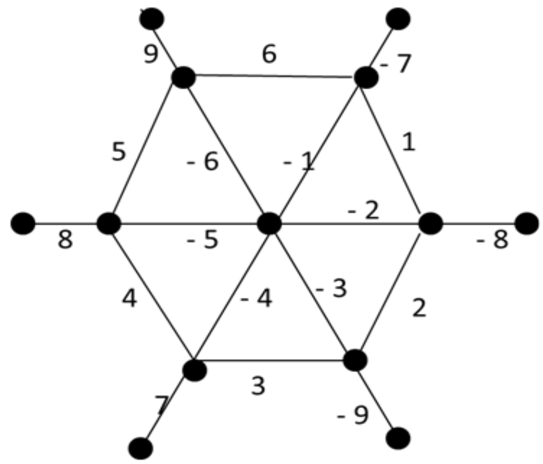

Figure 7.H_6 is integer edge cordial

Theorem 2.7

Flower graph Fl_n is integer edge cordial.

Proof

Let v be the apex vertex, $\{v_1, v_2, v_3, ..., v_n\}$ be the vertices of inner cycle; $\{u_1, u_2, u_3, ..., u_n\}$ be the rim vertices. Let $Fl_n = G$ then $p = 2n+1$ and $q = 4n$. To define $f : E \rightarrow [-\lfloor \frac{q}{2} \rfloor .. \lfloor \frac{q}{2} \rfloor]^*$ we consider the following labeling.

$f(vv_i) = -i; 1 \leq i \leq n$

$f(v_iu_i) = -(n+i); 1 \leq i \leq n$

$f(v_1v_n) = n$

$f(v_iv_{i+1}) = i; 1 \leq i \leq n$

$f(vu_i) = n+i; 1 \leq i \leq n$

Let us consider the vertex, $u_i; 1 \leq i \leq n$. Then $f(u_i) = f(u_iv_i) + f(vu_i) = -(n+i) + (n+i) = 0$

Hence we label as 1.

Also let us consider the vertex v_n. The vertex v_n is incident with the edge $vv_n, v_nv_1, v_nv_{n-1}, v_nu_n$.

We have, $f(vv_n) = -n; f(v_nv_1) = n$;$f(v_nv_{n-1}) = n-1; f(v_nu_n) = -2n$.

Now, $(v_n) = -n+n+n-1-2n = -(n+1)$, which is always negative.

Hence, $f(v_i); 1 \leq i \leq n$ receives label 0. From the above observation, $\lceil \frac{p}{2} \rceil$ vertices receives label 0 and $\lfloor \frac{p}{2} \rfloor$ vertices receive label 1. That is, $v_f^*(0) = \lceil \frac{p}{2} \rceil$ and $v_f^*(1) = \lfloor \frac{p}{2} \rfloor$

If n is even or odd we get $|v_f^*(0) - v_f^*(1)| = 1$.

Therefore, Fl_n is integer edge cordial as shown in figure 8.

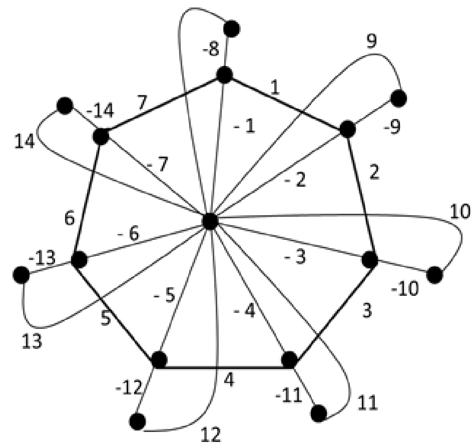

Figure 8.Fl_7 is integer edge cordial

Theorem 2.8

$K_{n,n}$ is not an integer edge cordial graph.

Proof

Let $G = K_{n,n}$ be a complete bipartite graph with the partitions $\{U,V\}$ where $U = \{u_1,u_2,u_3,...,u_n\}$ and $V = \{v_1,v_2,v_3,...,v_n\}$. Then $p = 2n$ and $q = n^2$

Let f be an integer edge cordial graph and let $f(u_iv_j) = e_j$ for all i, j

$$f(e_i) = \begin{cases} x_i & ; \quad 1 \leq i \leq \frac{n^2}{2} \\ -x_i & ; \quad \frac{n^2}{2} \leq i \leq n^2 \end{cases}$$

Now, $f(u_i)$; $1 \leq i \leq \frac{n}{2}$ receives positive integer and $f(u_i)$; $\frac{n}{2} \leq i \leq n$ receives negative integers. Also all the v_i vertices receives positive integer. That is, $(\frac{n}{2}+n)$ vertices receives label 1 and $\frac{n}{2}$ vertices receives label 0.

Hence, $v_f^*(0) = \frac{n}{2}$ and $v_f^*(1) = \frac{n}{2}+n$

Thus, $|v_f^*(0) - v_f^*(1)| = |\frac{n}{2}+n-\frac{n}{2}| = n \nleq 1$

Hence, $K_{n,n}$ is not an integer edge cordial graph.

The same proof will follow for the case, when n is odd.

Theorem 2.9

$K_{n,n} \backslash M$ where M is a perfect matching of $K_{n,n}$ is integer edge cordial if n is even.

Proof

Consider the graph $K_{n,n} \backslash M$ with vertex set $V(G) = \{X,Y\}$ where $X = \{x_1,x_2,x_3,...x_n\}$ and $Y = \{y_1,y_2,y_3,...,y_n\}$. Let $G = K_{n,n} \backslash M$ where M is the perfect matching of $K_{n,n}$.

Here $p = 2n$ and $q = n(n-1)$. We define $f : V \rightarrow [-\lfloor \frac{p}{2} \rfloor .. \lfloor \frac{p}{2} \rfloor]^*$ as follows:

Case (i) q is even (n even)

From the above labeling it is clear that $\frac{n}{2}$ vertices of the partite set X receive negative integers and $\frac{n}{2}$ vertices of Y receive negative integers and other vertices receive positive integers. That is, $\frac{n}{2}+\frac{n}{2} = n$ vertices receive neg-

y_j \ x_i	1	2	3	...	$\frac{n}{2}$	...	$n-1$	n
1	-	-1	-2	...	$-(\frac{n}{2}-1)$	...	$-(n-2)$	$-(n-1)$
2	-n	-	$-(n+1)$	...	$-(\frac{3n}{2}-1)$	...	$-[2(n-1)-1]$	$-2(n-1)$
3	$-(2n-1)$	$-2n$	-	...	$-(\frac{5n}{2}-1)$	...	$-[3(n-1)-1]$	$-3(n-1)$
...	...	...	...	...	...	...	...	...
$\frac{n}{2}$	$-[\frac{(n-1)(n-2)}{2}+1]$	$-[\frac{(n-1)(n-2)}{2}+2]$	$-[\frac{(n-1)(n-2)}{2}+3]$	...	-	$\dots$	$-[\frac{n}{2}(n-1)-1]$	$-\frac{n}{2}(n-1)$
$\frac{n}{2}+1$	1	2	3	...	$\frac{n}{3}$	...	$n-2$	$n-1$
...	...	...	...	...	...	...	...	...
n	$\frac{(n-1)(n-2)}{2}+1$	$\frac{(n-1)(n-2)}{2}+2$	$\frac{(n-1)(n-2)}{2}+3$	...	$\frac{(n-1)(n-2)}{2}+n$	...	$\frac{n}{2}(n-1)$	-

ative integers and hence label 0. Similarly, n vertices receive label 1.

Therefore, $v_f^*(0) = v_f^*(1) = \frac{p}{2}$

Hence $|v_f^*(0) - v_f^*(1)| = 0$

When q is odd (n odd)

Assume $K_{n,n} \backslash M$ to be an integer edge cordial.

Then among (n - 1) vertices of X, $\frac{(n-1)}{2}$ vertices receives label 0 and $\frac{(n-1)}{2}$ vertices receives label 1.

Similarly in Y, $\frac{(n-1)}{2}$ vertices receive label 0 and 1. And one vertex from each X and Y partitions receive label 1. That is $v_f^*(1) = \frac{(n-1)}{2} + 2$ and $v_f^*(0) = \frac{(n-1)}{2}$.

Hence $|v_f^*(0) - v_f^*(1)| \nleq 1$.

Thus $K_{n,n} \backslash M$ is integer edge cordial if n is even.

REFERENCES

[1] I.Cahit, Cordial Graphs, A Weaker Version of Graceful and Harmonious Graphs, Ars Combinatoria, Vol. 23, 1987, pp. 201-208.

[2] I.Cahit, Contemporary Methods in Graph Theory, Wissenschaftsverlog, Mannheim, 1990, pp. 209-230.

[3] I.Cahit, H- Cordial Graphs, Bulletin of the Institute Combinatorics and its Applications, Vol. 18, 1996, pp. 87-101.

[4] J.Devaraj, On Edge-Cordial Graphs, Graph Theory Notes of New York, 2004, pp. 14-18.

[5] G.M. Du, Cordiality of Complete K- Partite Graphs and Some Special Graphs, Neimenggu Shida Xuebao Ziran Kexue Hauwen Ban, Vol. 2, 1997, pp. 9-12

[6] J.A.Gallian, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics, Vol. 5, 2000, pp. 1-79.

[7] F.Harary, Graph Theory, Addison- Wesly, Reading Mass, USA, 1972, pp. 8-202.

[8] Archit Yajnik and Rustam Ali, Numerical Solution of Integral Equations by using Discrete GHM Multi-wavelet, DJ Journal of Engineering and Applied Mathematics, Vol. 1, No. 1, 2015, pp. 17-22, https://dx.doi.org/10.18831/djmaths.org/2015011003.

[9] M.Hovey, A-Cordial Graphs, Discrete Mathematics, Vol. 93, No. 2-3, 1991, pp. 183-194, https://dx.doi.org/10.1016/0012-365X(91)90254-Y.

[10] M.Sundaram, R.Ponraj and S.Somu Sundaram, Prime Cordial labeling of graphs, Journal of Indian Academy of Mathematics, Vol. 27, 2005, pp. 373-393.

[11] A.Unveren and I.Cahit, M-Cordial Graphs, Pre-print.

[12] Mehar Chand, Sequence of Functions Involving the Product of S-Functions, DJ Journal of Engineering and Applied Mathematics, Vol. 2, No. 2, 2016, pp. 15-23, https://dx.doi.org/10.18831/djmaths.org/2016021003.

[13] R.Varatharajan, S.Navaneetha Krishnan and K.Nagarajan, Divisor Cordial Graphs, International Journal of Mathematical Combinatorics, Vol. 4, 2011, pp. 15 – 25.

[14] R.Yilmaz and I.Cahit, E-Cordial Graphs, Ars Combinatoria, Vol. 46, 1997, pp. 251-266.