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RESEARCH ARTICLE

Cascade and System Reliability for Exponential Distributions

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ABSTRACT

This paper presents the estimation of $R = P(X > Y)$ by considering cascade stress-strength model. The system survives if and only if the strength (X) is greater than the stress (Y), otherwise fails. The statistical model is formed to study cascade and system reliability. Here, we assume that all the components are independent and follow the one-parameter exponential strength-stress distributions. The first four cascade reliabilities are computed for different values of strength-stress parameters. The statistical variation in cascade and system reliability is shown graphically.

Keywords: Attenuation factor, Cascade reliability, Exponential distribution, Statistical model, Strength-stress model.

1. INTRODUCTION

Cascade means; a small waterfall, typically one of several that fall in stages down a steep rocky slope; 'the waterfall raced down in a series of cascades; A large number or amount of something occurring at the same time; A process whereby something, typically information or knowledge, is successively passed on.

Reliability is the probability of success or the probability that the system will perform its intended function under specified design limits. More specifically, reliability is the probability that a product or part will operate properly for a specified period of time (design life) under the design operating conditions (such as temperature, volt, etc.) without failure. In other words, reliability may be used as a measure of the system's success in providing its function properly. Mathematically, reliability R(t) is the probability that a system will be successful in the interval from time 0 to time $t : R(t) = P(T > t)$ for $t \geq 0$, where T is a random variable denoting the time-to-failure or failure time.

The exponential distribution is often used to model the reliability of electronic systems, which do not typically experience wearout type failures. The exponential distribution probability density function, reliability function and hazard rate are given by:

$$f(t) = \lambda e^{-\lambda t} \quad \textit{Probability Density Function}$$

$$R(t) = \int_t^\infty f(t)dt = e^{-\lambda t} \quad \textit{Reliability Function}$$

$$h(t) = \frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{\lambda e^{-\lambda t}} = \lambda \quad \textit{Hazard Rate}$$

Cascade systems were first developed and studied by [1]. They have studied an n-cascade reliability for exponential distribution. They computed reliability values for a 2-cascade system with gamma and normal stress and strength distributions. [2] studied the reliability of a cascade system with normal stress and strength distribution. [3] Rekha and Shyam Sundar have derived an expression of the reliability of an n-cascade system by considering attenuation factor with the same parameter value. For their study, they considered exponential strength and gamma stress distributions. [4] has done a case study of cascade reliability using Weibull distribution. [5] have studied cascade reliability of stress-strength system considering strength follows mixed exponential distribution. [6] has done

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comparison of an n-cascade system by considering normal stress and exponential strength distribution. [7] studied the reliability analysis of a redundant cascade system by using Markovian approach assuming n-components which are arranged in the hierarchial order. The n-cascade system survive with loss of m components by k number of attacks [7]. [8] studied about estimation of reliability for stress-strength cascade model by comparison between estimators made using data obtained through simulation experiment [8, 9, 10].

Let the random variable X denotes the strength and another random variable Y denotes stress. Then the reliability of the component is given by

$$\begin{aligned} R = P(X > Y) &= \int_0^\infty \left(\int_y^\infty f(x)dx \right) g(y)dy \\ &= \int_0^\infty \bar{F}(y)g(y)dy \end{aligned} \tag{1}$$

where f(x) and g(y) are probability density functions of X and Y respectively. F(y) is the distribution function of Y and $\bar{F}(y) = 1 - F(y)$.

The reliability is the measure of the system performance such that the system survives if strength exceeds stress; otherwise, system fails at any time. Cascade reliability model is a special type of stress- strength model. In an n-cascade system, it is assumed that each component faces changed stress imposed on it, while taking the place of failed component. The system survives up to the last component works. If all the components in cascade fail then system fails. This changing stress is calculated by product of K and stress on previous component where K is an integer constant known as the attenuation factor. The purpose of this work is to study statistical variation in cascade and system reliability using exponential strength-stress distribution.

This paper formulates a statistical model for exponential strength-stress distributions which are considered for computations of marginal and system reliabilities supported by reliability computations with numerical and graphical study along with various observations.

2. STATISTICAL MODEL

Let $X_1, X_2, ...X_n$ be the set of n independent random variables which denotes the strength of the components arranged in the order of activation respectively, having probability density function Let another random variable Y be the stress imposed on the n components, having probability density function The system survives up to failure of first (n-1) components i.e. $X_i < Y; i = 1, 2, ..., (n-1)$ and $X_n > Y$. The system reliability R_n is given by

$$R_n = R(1) + R(2) + ... + R(n) \tag{2}$$

The marginal reliability R(n) which is the reliability of the system for the n^{th} component is given by,

$$R(n) = p\left[X_1 < Y, X_2 < Y, ...X_{n-1} < Y, X_n > Y\right] \tag{3}$$

Hence,

$$R(n) = \int_0^\infty \left[\int_0^y f_1(x_1)dx_1 \times \int_0^y f_2(x_2)dx_2 \times ... \times \int_0^y f_{n-1}(x_{n-1})dx_{n-1} \times \int_0^\infty f_n(x_n)dx_n \right] g(y)dy$$

$$R(n) = \int_0^\infty F(y).F(y)...F(y)\bar{F}(y)g(y)dy \tag{4}$$

where,

$$F_i(y) = \int_0^y f_i(x_i)dx_i, \quad i = 1, 2, ..., n$$

and

$$\bar{F}_i(y) = 1 - F_i(y).$$

3. RELIABILITY COMPUTATION

In the following section, we compute reliability for one parameter exponential strength and one parameter exponential stress distribution. Equations (1) to (9) shows the analysis.

Let random variable X (strength) follows exponential distribution with parameter a. The probability density function of X is given by,

$$f(x;a) = aexp(-ax), \quad x > 0, \quad a > 0 \tag{5}$$

Its distribution function is given by,

$$F(x) = 1 - exp(-ax) \tag{6}$$

Let another random variable Y (stress) follows exponential distribution with parameter b . The probability density function of Y is given by

$$g(y;b) = bexp(-by), \quad y > 0, \quad b > 0 \tag{7}$$

$$G(y) = 1 - exp(-by) \tag{8}$$

Then from (4), we calculate cascade reliability.

i) 1- cascade reliability,

$$R(1) = P(X_1 > Y) = \int_0^\infty \bar{F_1}(y)g(y)dy$$
$$= \int_0^\infty e^{-ay}be^{-by}dy = \frac{b}{a+b}$$

ii) 2-cascade reliability,

$$R(2) = P(X_1 > Y, X_2 > Y)$$
$$= \left[\int_0^\infty F_1(y)\bar{F_2}(y)g(y)dy\right]$$
$$= \int_0^\infty (1-e^{-ay})e^{-ay}be^{-by}dy$$
$$= \frac{ab}{(a+b)(2a+b)}$$

Similarly,

iii) 3-cascade reliability,

$$R(3) = \frac{2a^2b}{(a+b)(2a+b)(4a+b)}$$

iv) 4-cascade reliability,

$$R(4) = \frac{6a^2b}{(a+b)(2a+b)(3a+b)(4a+b)}$$

In general, an n-cascade reliability,

$$R(n) = \frac{(n-1)!a^{n-1}b}{\Pi_{i=1}^{n}(ia+b)} \tag{9}$$

4. NUMERICAL AND GRAPHICAL STUDY

In the following section, the numerical and graphical study by calculating cascade and system reliability is carried out. Considering the first set strength parameter as constant i.e. 4 and then progressively increasing it i. e. from 1 to 10 where as the stress parameter is progressively increased i.e. from 1 to 10 and then made constant i.e. 4, the scenario as shown in table 1 is obtained. Similarly, we set the strength parameter progressively increasing i.e. from 1 to 10, then decreasing i.e. from 10 to 1 whereas the stress parameter is progressively decreasing i.e. from 10 to 1 then increasing i.e. from 1 to 10, as shown in table 2. Now we calculate the cascade reliabilities using (9).Further we compute the system reliability R_4 using (2). Next, we plot graphs by taking the progressive values of strength and stress versus cascade and system reliability respectively. Here note that R(1)=R1.

Table 1.Cascade and system reliability

a	b	R(1)	R(2)	R(3)	R(4)	R_4
4	1	0.2	0.088889	0.054701	0.038612	0.382202
4	2	0.333333	0.133333	0.07619	0.050794	0.593651
4	3	0.428571	0.155844	0.083117	0.052495	e0.720027
4	4	0.5	0.166667	0.083333	0.05	0.8
4	5	0.555556	0.17094	0.080442	0.045967	0.852905
4	6	0.6	0.171429	0.07619	0.041558	0.889177
4	7	0.636364	0.169697	0.071451	0.037279	0.914791
4	8	0.666667	0.166667	0.066667	0.033333	0.933333
4	9	0.692308	0.162896	0.062056	0.029787	0.947046
4	10	0.714286	0.15873	0.05772	0.02664	0.957376
1	4	0.8	0.133333	0.038095	0.014286	0.985714
2	4	0.666667	0.166667	0.066667	0.033333	0.933333
3	4	0.571429	0.171429	0.079121	0.044505	0.866484
4	4	0.5	0.166667	0.083333	0.05	0.8
5	4	0.444444	0.15873	0.083542	0.052214	0.738931
6	4	0.4	0.15	0.081818	0.052597	0.684416
7	4	0.363636	0.141414	0.079192	0.05197	0.636212
8	4	0.333333	0.133333	0.07619	0.050794	0.593651
9	4	0.307692	0.125874	0.073088	0.049335	0.555989
10	4	0.285714	0.119048	0.070028	0.047746	0.522536

Table 2.Cascade and system reliability

a	b	R(1)	(2)	R(3)	(4)	R_4
1	10	0.909091	0.075758	0.011655	0.002498	0.999001
2	9	0.818182	0.125874	0.033566	0.011847	0.989469
3	8	0.727273	0.155844	0.055004	0.024752	0.962872
4	7	0.636364	0.169697	0.071451	0.037279	0.914791
5	6	0.545455	0.170455	0.081169	0.046828	0.843906
6	5	0.454545	0.160428	0.083701	0.051953	0.750627
7	4	0.363636	0.141414	0.079192	0.05197	0.636212
8	3	0.272727	0.114833	0.068049	0.046662	0.502271
9	2	0.181818	0.081818	0.050784	0.036083	0.350503
10	1	0.090909	0.04329	0.027929	0.020436	0.182564
9	2	0.181818	0.081818	0.050784	0.036083	0.350503
8	3	0.272727	0.114833	0.068049	0.046662	0.502271
7	4	0.363636	0.141414	0.079192	0.05197	0.636212
6	5	0.454545	0.160428	0.083701	0.051953	0.750627
5	6	0.545455	0.170455	0.081169	0.046828	0.843906
4	7	0.636364	0.169697	0.071451	0.037279	0.914791
3	8	0.727273	0.155844	0.055004	0.024752	0.962872
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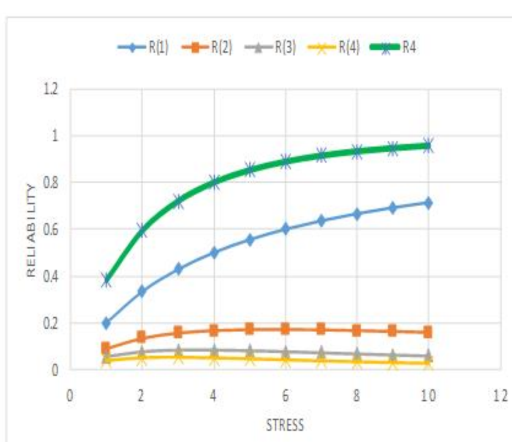

Figure 1.Cascade and system reliability (for constant strength parameter ‘a’)

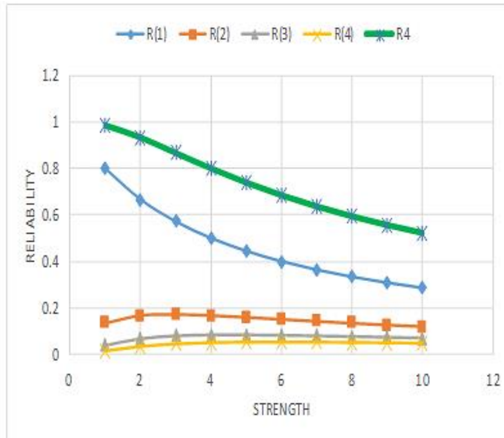

Figure 2.Cascade and system reliability (for constant stress parameter ‘b’)

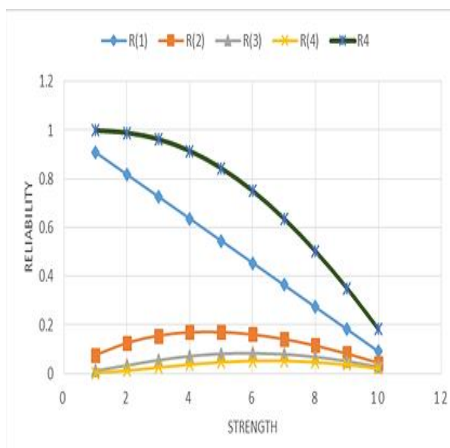

Figure 3.Cascade and system reliability (for increasing strength parameter ‘a’)

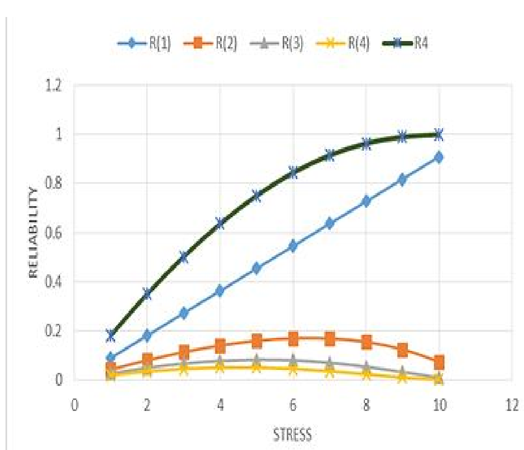

Figure 4.Cascade and system reliability (for increasing stress parameter ‘b’)

5. OBSERVATIONS/DISCUSSIONS

The following observations are made from the above tables and figure 1, figure 2, figure 3 and figure 4.

1. If strength parameter is constant and stress parameter increases, the cascade reliability R(1) increases whereas R(2), R(3) and R(4 increases at the beginning and then decreases. But the system reliability R_4 is increasing. In table 1, system reliability increases from 0.382202 to 0.957376.
2. If strength parameter increases and stress parameter is a constant, the cascade reliability R(1) increases whereas R(2), R(3) and R(4) increases at the beginning and then decreases. But the system reliability R_4 is increasing. In table 1, system reliability decreases from 0.985714 to 0.522536.
3. In figure 1, the cascade reliability R(1) is far away from R(2), R(3) and R(4). As stress parameter increases, there is a rapid change in R_4 and reaches near unity.
4. In figure 2, the cascade reliability R(1) decreases but there is less variation in R(2), R(3) and R(4). As strength parameter increases, the system reliability R_4 rapidly decreases.
5. If strength parameter increases and stress parameter decreases, the cascade reliability R(1) decreases whereas R(2), R(3) and R(4) increases at the beginning and then decreases but the system reliability R_4 is decreasing. In table 2, system reliability decreases from 0.999001to 0.182564.
6. If strength parameter decreases and stress parameter increases, the cascade reliability R(1) increases whereas R(2), R(3) and R(4) increases at the beginning and then decreases. But the system reliability R_4 is increasing. In table 2, system reliability increases from 0.182564 to 0.999001.

6. CONCLUSION

In this paper, the reliability formula for an n- cascade system using exponential strength-stress distributions is found out. We observe that the system reliability R4 reaches near to unity if we keep strength parameter constant and stress parameter progressively increasing. We also came across the finding that the system reliability R4 rapidly decreases if we keep stress parameter constant and strength parameter progressively increasing. In addition, if we increase strength parameter and decrease stress parameter, the cascade reliability little bit increases and then decreases but system reliability rapidly decreases. Again, if we decrease strength parameter and increase stress parameter, the cascade reliability little bit increases and then decreases but system reliability rapidly increases and reaches to unity. Here, it is to be noted that the cascade and system reliability is found to be constant if strength and stress parameters are progressively increasing.

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