DJ Journal of Engineering and Applied Mathematics, Vol. 2(1) 2016, pp. 7-11

RESEARCH ARTICLE

Modelling of two Interconnected Spring Carts and Minimization of Energy

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Received-10 December 2015, Revised-10 January 2016, Accepted-20 January 2016, Published-31 January 2016

ABSTRACT

Focus is to form a mathematical model for interconnected spring carts to minimize the energy using Powell's method of minimization. Expressing the energy of a surface in terms of its local deformation, local force on the surface is given by differentiating the energy with respect to position that gives an equation. The modelled equations make a system that has been converted in to a biquadratic function and used to minimize the energy to save the life of spring in the best method of optimization. The Powell's method is given in detail and compared with Downhill Simplex method.
Keywords: Mathematical modelling, Interconnected spring carts, Ordinary differential equation, Derivative-free optimization, Powell's method.

1. INTRODUCTION

Many processes in mechanical industry hold instable behaviour with several types of reactors and instruments. Modelling and simulation is the only framework to examine properties of these mechanical systems. In recent years, the optimization for the solution of engineering models has evolved. For optimization, direct search methods are very suitable for simple engineering problems involving a relatively small number of variables. In the optimization area, there are many effective approximation methods to simulate engineering problems [1, 2]. Most widely used Powell's conjugate direction method, is suitable for computing local minimum value of a complex function without taking derivatives [3]. Powell's method is a derivative-free technique widely used as a direct search method to optimize quadratic functions in a finite number of steps to have a better convergent [4].

From a simulation point of view the mass spring models are efficient models, where Newton's second law is the principle used to generate the mathematical representation of a spring carts system. The spring carts system is modelled both as a single degree of freedom system and a multi degree of freedom system for the study of the mechanical system. Our model is based on the physical performance of interconnected spring carts system where, when springs are stretched, they exert a force that requires energy management in an efficient way to increase the life of the spring [5]. Unique carts system allows the wheels to interact inside the cart to prevent damage to the internal components of the cart. Interconnected spring carts system is modelled as a multi-degree-of-freedom system to facilitate vibration behaviour [6]. There are many methods in differential calculus to construct mathematical equations for single and multi-degree of freedom like physical lumping, finite element method, finite difference method and Galerkin where the behaviour of dynamical elements in multi-degree-of-freedom has been modelled as a system of ordinary differential equations to make it a quadratic polynomial [7, 8, 9, 10].

The main objectives of this work are to model and optimize the energy of spring carts system so that the life of the spring is saved and to use the best appropriate method for this type of mathematical model. For this purpose we have considered a system of carts

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Double blind peer review under responsibility of DJ Publications
http://dx.doi.org/10.18831/djmaths.org/2016011002

connected with springs at a uniform condition supporting the basic elements of spring obeying hook's law and Newton's law.

2. MATHEMATICAL MODELLING OF CART SPRING SYSTEM

A very low friction cart design offers a smooth motion and is used to generate a mathematical model for connected springs. The system consists of a cart which is connected with a spring and the major quantities used to describe the dynamical behaviour of the system are mass, time, distance and force, where the basic principles are hook's and newton's laws of motion. The model is constructed by two carts having masses of v_1 and v_2 connected to springs of spring constants a_1 and a_2 where the assumption is that cart-1 is connected to a fixed spring of spring constant a_1 and cart-2 is connected with cart-1 by a spring of spring constant a_2. δ_i is the difference between the spring current length and its given length. Damping free mathematical representation of cart-1 is given in equations (2.1) and (2.2)

$$f_1 = a_1\delta_1 - a_2(\delta_2 - \delta_1) \quad (2.1)$$

$$v_1 \frac{d^2}{dt^2}\delta_1 = a_1\delta_1 - a_2(\delta_2 - \delta_1) \quad (2.2)$$

As the second cart is connected with cart-1 by a spring of spring constant a_2, the mathematical representation of cart-2 with zero damping is given by equations (2.3) and (2.4)

$$f_2 = a_2(\delta_2 - \delta_1) \quad (2.3)$$

$$v_2 \frac{d^2}{dt^2}\delta_2 = a_2(\delta_2 - \delta_1) \quad (2.4)$$

Equations (2.2) & (2.4) can be written in matrix form as shown in (2.5)

$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 & -a_2 \\ -a_2 & a_2 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \quad (2.5)$$

For Powell's implementation, the function must be a real valued function. So we have to generate the bi-variate quadratic model in second-degree polynomial that has the form

$$f(\delta_i) = \frac{1}{2}\varepsilon^* A\varepsilon - \varepsilon^* A \quad (2.6)$$

where ε = A matrix of δ_i

ε^* = The transpose of ε

A = Matrix of spring constant.

Replacing these matrices from equation (2.5) in equation (2.6), we get the spring cart system shown in (2.7)

$$f(\delta_1, \delta_2) = b_1\delta_1^2 - b_2\delta_1\delta_2 + b_3\delta_2^2 - b_4\delta_1 - b_5\delta_2 \quad (2.7)$$

3. FORMULATION

For the effective usage of developed mathematical model in ideal circumstances using uniform behaviour of carts spring system, we use approximate values based on the dynamical behaviour of the cart system.

It is assumed:

$$a_1 = 100, a_2 = 500, f_1 = 200, f_2 = 200$$

Using this data in equation (2.5) and (2.6) we have the final shape of equation (2.7) given by equation (3.1)

$$f(\delta_1, \delta_2) = 300\delta_1^2 - 500\delta_1\delta_2 + 250\delta_2^2 - 200\delta_1 - 300\delta_2 \quad (3.1)$$

4. POWELL'S METHOD

This technique minimizes the quadratic function as follows: 1 point is assumed to generate two vectors. Based on the linear combination of vectors a new point is generated to minimize the function.

Having the initial difference (1, 1) the bi-variate quadratic function given in (3.1) is minimized in each direction starting with the second coordinate direction. Similarly for the succeeding cycle of minimization, we discard single earlier used coordinate direction to have the newly generated pattern until the required pattern is achieved. Repeating this process, after little iteration, we get the values of unknowns that minimize the quadratic function in (3.1).

5. DOWNHILL SIMPLEX

In order to calculate the reflected point, we accept the reflected point better than the next and then get a next simplex by substituting the reflected point. Idea is to employ a moving simplex method in the space design for surrounding the minimum point to shrink the simplex until the dimensions get acceptance error. This method approaches a local optimal of bi-variate quadratic function (3.1) of spring cart system after 46 iterations. Comparing the Downhill Simplex and Powell's method, it is seen that Downhill simplex is much slower than Powell's method which is depicted in table A1.

6. CONCLUSION

The aim of this project was to model and optimize the interconnected carts by using the best optimization techniques. We have modelled the system of differential equations and discussed two different methods of optimization viz Powell's method & Simplex method to optimize the energy function of carts spring system. Matlab data of interconnected spring carts for minimization of energy function by using Powell's and Downhill Simplex methods is shown in figure B1 and B2. The surface plot in figure B1 represents the minimum difference that is 5.0 and 5.6 units between the spring's carts. Figure B2 represents the simple contour where the function has minimum values that are represented in table A1. These values show that Powell's method is much better than Downhill Simplex.

REFERENCES

[1] M.J.D.Powell, An Efficient Method for Finding the Minimum of a Function of Several Variables Without Calculating Derivatives, The Computer Journal, Vol. 7, No. 2, 1964, pp. 155-162, http://dx.doi.org/10.1093/comjnl/7.2.155.

[2] M.J.Thoresson, P.E.Uys, P.S.Els and J.A.Snyman, Efficient Optimization of a Vehicle Suspension System, Using a Gradient-Based Approximation Method, Part 1: Mathematical Modelling, Mathematical and Computer Modelling, Vol. 50, No. 9-10, 2009, pp. 1421-1436, http://dx.doi.org/10.1016/j.mcm.2009.07.011.

[3] M.J.D.Powell, Beyond Symmetric Broyden for Updating Quadratic Models in Minimization without Derivatives, Mathematical Programming, Vol. 138, No. 1, 2013, pp. 475-500, http://dx.doi.org/10.1007/s10107-011-0510-y

[4] Tamara G.Kolda, Robert Michael Lewis, Virginia Torczon, Optimization by Direct Search: New Perspectives on some Classical and Modern Methods, SIAM Review, Vol. 45, 2003, pp. 385–482.

[5] Jose Francisco Gomez-Aguilar, Huitzilin Yepez-Martinez, Celia Calderon-Ramon, Ines Cruz-Orduna, Ricardo Fabricio Escobar-Jimenez and Victor Hugo Olivares-Peregrino, Modelling of a Mass-Spring-Damper System by Fractional Derivatives with and without a Singular Kernel, Entropy, Vol. 17, No. 9, 2015, pp. 6289-6303, http://dx.doi.org/10.3390/e17096289

[6] W.T. Thomoson and M.D. Dahleh, Theory of Vibration with Applications. Fifth Edition, Prentice Hall, New Jersey, 1998.

[7] J.Rahman, M.Mushtaq, A.Ali, Y.N. Anjam and S.Nazir, Modelling Damped Mass-Spring System in Matlab Simulink, Journal of Faculty of Engineering and Technology, Vol. 21, No. 2, 2014, pp. 21-28.

[8] Jamshaid ul Rahman, Muhammad Raheel Mohyuddin, Naveed Anjum and Syed Zahoor, Mathematical Modelling and Simulation of Mixing of Salt in 3-Interconnected Tanks, Journal of Advances in Civil Engineering, Vol. 1, No. 1, 2015, pp. 1-6, http://dx.doi.org/10.18831/djcivil.org/2015011001

[9] Jamshaid ul Rahman, Muhammad Raheel Mohyuddin, S.V Satyanarayana, Syed Zahoor and Amira Al-Ghareebi, Mathematical and Simulink Model for Paint Industry Effluent, DJ Journal of Engineering Chemistry and Fuel, Vol. 1, No. 1, 2016, http://dx.doi.org/10.18831/djchem.org/2016021001

[10] M.J.D. Powell, Least Frobenius Norm Updating of Quadratic Models that Satisfy Interpolation Conditions, Mathematical Programming, Vol. 100, No. 1, 2004, pp. 183-215, http://dx.doi.org/10.1007/s10107-003-0490-7.

APPENDIX A

Table A1.Powell's Method to Downhill Simplex Method

Sr.	Algorithm	Iterations	δ -Min	f-Min
1	Powell's	3	$\delta_1 = 5.0000$ $\delta_2 = 5.6000$	-1.3400e+003
2	Simplex	46	$\delta_1 = 5.0000$ $\delta_2 = 5.6000$	-1.3400e+003

APPENDIX B

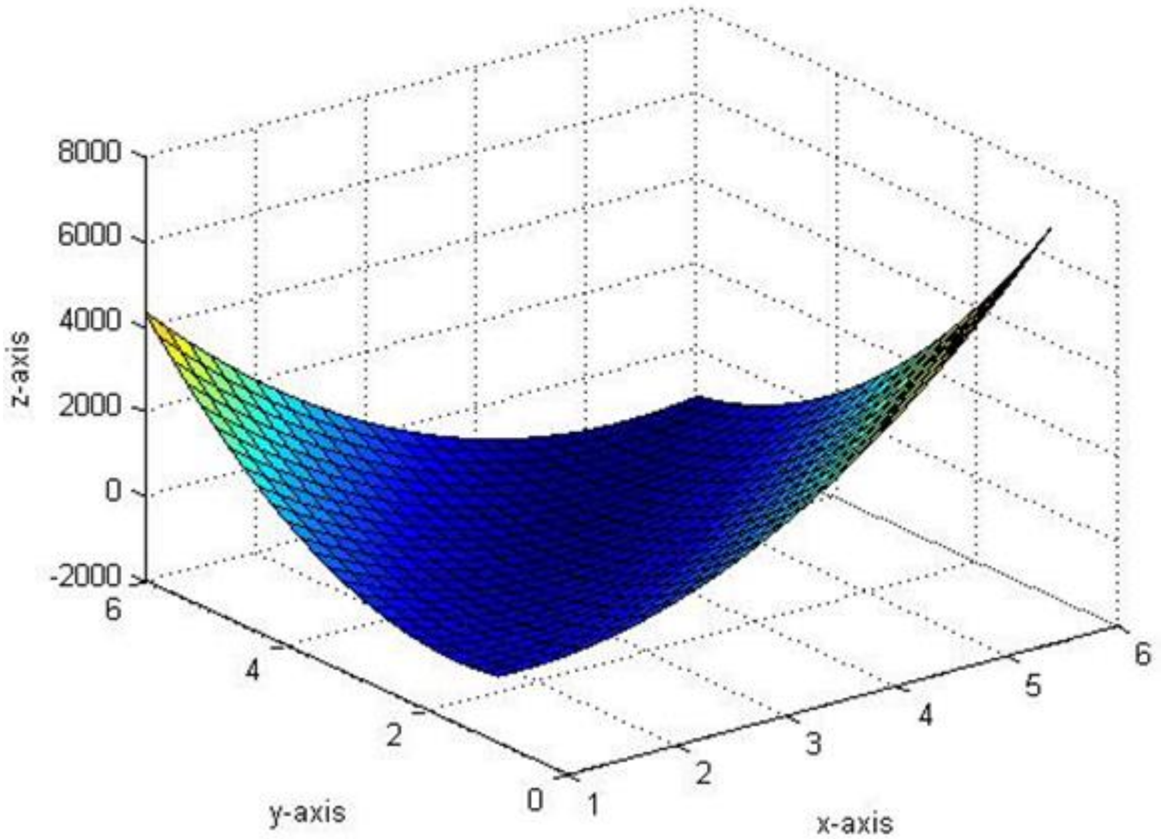

Figure B1.Minimization of difference between spring carts

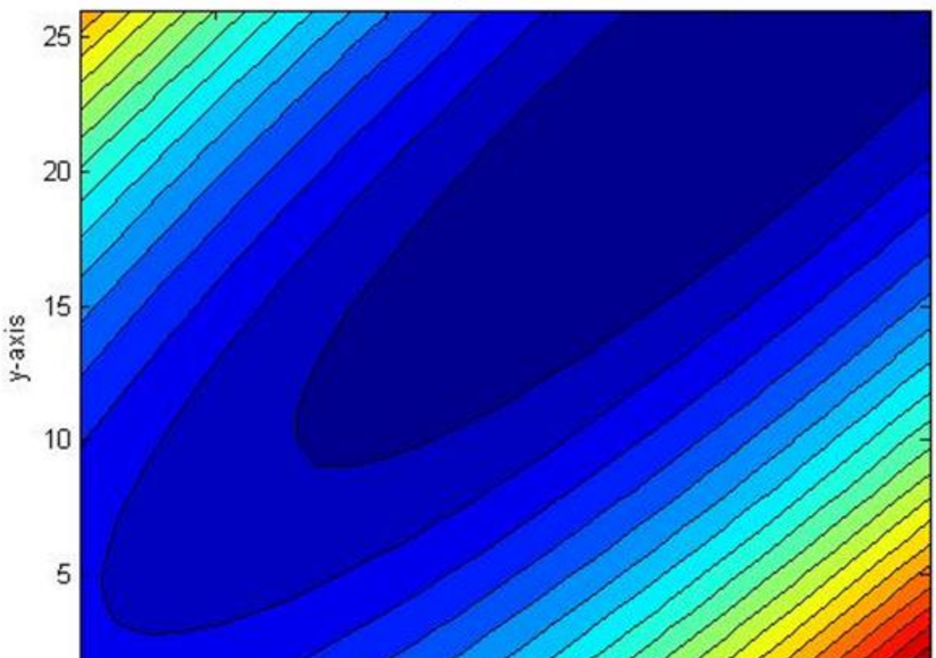

Figure B2.Simple contour