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RESEARCH ARTICLE

Subdivided Stars Super (b, d) - Edge-Antimagic Total Graph Labelling

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ABSTRACT

In the existing method the authors assume that there admits in every tree a labeling of edge-magic total. The existing author proposed the speculations that every tree is a super (b, d)- graph of edge-antimagic total.In this proposed paper, we frame the subdivided star super (b, d)-edge-antimagic total labeling $T\ (n, n, n+4, n+4, n_5, n_6..., n_q)$ for d ∈ {0, 1, 2}, where q ≥ 5, $n_s = 2s - 4\ (n + 3) + 1$, 5 ≤ s≤ q and n ≥ 3 is odd.

Keywords: Vertex-set, Edge-set, Total labeling, Magic constant and Trees of subclass.

1. INTRODUCTION

In this paper all graphs are limited, simple and undirected. For a graph G, V (G) and E (G) denote the vertex-set and the edge-set, respectively. A (v, e)-graph G is a graph such that |V (G)| = v and |E (G)| = e. A general reference for graph-theoretic ideas can be seen in [1]. A labeling (or valuation) of a graph is a map that carries graph elements to numbers (usually to positive or non-negative integers). In this paper, the domain will be the set for all vertices and edges and such a labeling is called a total labeling. Some labelings use the vertex-set only or the edge-set only and we shall call them vertex-labelings or edge-labelings, respectively. A number of classification studies on edge-antimagic total graphs have been intensively investigated. For more detailed study on antimagic labelings, see [2].

Description 1.1

A (p, d)-edge-antimagic vertex ((p, d)-EAV) a graph G labeling is a function of bijective λ: $V(G) \rightarrow \{1, 2, \dots, v\}$ such that all edges in G the set of edge-sums,$\{u(x\,y) = \lambda(x) + \lambda(y) : x\,y \in E(G)\}$, forms an additive series $\{p, p + d, p + 2d, \dots, p + (e - 1)d\}$, where p > 0 and d ≥ 0 are two static integers.

Description 1.2

A (b, d)-edge-antimagic total ((b, d)-EAT) a graph G labeling is a bijective function $\lambda : V(G) \cup E(G) \rightarrow \{1, 2, \dots, v + e\}$ such that all edges in G the set of edge-sums , $\{u(x\,y) = \lambda(x) + \lambda(y) : x\,y \in E(G)\}$, forms an additive series $\{b, b + d, b + 2d, \dots, b + (e - 1)d\}$, where b > 0 and d ≥ 0 are two static integers. If such scenario exists in labeling then G is said to be a (a, d)-EAT graph.

Description 1.3

A (b, d)-EAT labeling λ is called a super (b, d)-edge-antimagic total (super (b, d)-edge-antimagic total) G labeling if $\lambda\,(V\,(G)) = \{1, 2, \dots, v\}$. Thus, a super (b, d) - edge-antimagic total graph is a graph that acknowledges a super (b, d) - labeling of edge-antimagic total. In the previous definition, for d = 0, a super (b, 0) - labeling of edge-antimagic total is called a labeling in super edge-magic total (SEMT) and a is called a constant of magic. Furthermore, for d ≠ 0, a is called the edge-weight of minimum. An edge-magic total labeling subject of graphs has its origin in the [3] works on what they called ratings in graphs magic. A (b, d) - edge-antimagic total labeling definition was presented by [4] as an edge-magic total labeling natural extension defined by Kotzig. A super (b, d)- labeling of edge-antimagic total is a natural extension of the notion super (b, 0)-

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labeling of edge-antimagic total introduced by [5]. They also proposed the following speculation:

Speculations 1.1

Every tree admits a super (a, 0) - labeling of edge-antimagic total [6]. In this speculation, authors have considered a super (a, d) - labeling of EAT for many specific tree classes which can be detected in [7]. [8] verified these speculations by a tree's computer search with at most 17 vertices. However, these speculations are still open. Let us consider the proposition which is following for often use in the main effects.

Statement 1.1

[9] If b (v, e) - G graph has a (p, d) - labeling of edge-antimagic vertex then

(i) G has a super $(p + v + 1, d + 1)$ - labeling of edge-antimagic total.

(ii) G has a super $(p + v + e, d - 1)$ - labeling of edge-antimagic total.

The dual labeling notion has been presented by [10]. The lemma follows from the duality principal, which is review by [11].

Supporting theorem 1.1

If h is a super (b, 0) - G edge-antimagic total labeling with the constant of magic a, then the function g′: V (G) ∪ E (G)$\rightarrow\{1, 2, \ldots, v+e\}$ can be defined using equation (1.1).

$$g'(x) = \begin{cases} v + 1 - g(x), & \text{for } x \in V(G), \\ 2v + e + 1 - g(x), & \text{for } x \in E(G), \end{cases} \qquad (1.1)$$

It is also a super (b′, o)- G edge-antimagic total labeling with the constant of magic $b' = 4v + e + 3 - b$.

2. MAIN END

In this unit, we explain the subdivided stars notion and present some results of known different subdivided stars subclasses. In the end, we state some supporting theorems related to bounds of lower and upper areas of the parameters in antimagic labelling for the subdivided stars diverse subclasses.

Description 2.1

For $n_i \geq 1$, $q \geq 2$ and $1 \leq i \leq q$, let T (n1, n2, . . . , nq) be a star of subdivided classes obtained by inserting vertices of $n_i - 1$ to the i^{th} edge of the star K1,q . Thus, the subdivided star

$T\underbrace{(1, 1, \ldots, 1)}_{r\text{-times}}$ is the star K_1, q. A star of a subdivided class is a tree's specific class and many authors have proved the subdivided stars antimagicness. Some of the known results are as follows:

- [12, 13] called the star of subdivided T (m, n, k) as a three-path tree and proved that it is a super (b, 0)-EAT if m and n are odd functions with $k = n + 1$ or $k = n + 2$. [12] Verified that T (m, n, k) also admits a super (b, 0)- labeling of edge-antimagic total if m and n are odd functions with $k = n + 3$ or $k = n + 4$.
- [10] proved the existence of a super (b, o)-labeling of EAT on the star of subdivided classes denoted by S_n^m for m = 1, 2, where

$$S_n^1 \cong T\underbrace{(2, 2, \ldots, 2)}_{n\text{-times}} \quad \text{and} \quad S_n^2 \cong T\underbrace{(3, 3, \ldots, 3)}_{n\text{-times}}$$

- [11], created the subdivided star subclasses T (n1, n2, n3, n4) and derived a super (b, 0)-edge-antimagic total labeling existence on them. In the similar paper, certain results related to w-trees subdivision are also verified. [13] derived the bounds of lower and upper areas of the parameter labeling of antimagic graphs related to the most common subdivided stars subclass denoted by T $(n_1, n_2, n_3, \ldots, n_q)$ for any $n_i \geq 1$. Furthermore, subclasses of different trees are derived to be super (b, d) - edge-antimagic total under some conditions.

However, super (b, d)- labeling of edge-antimagic total of T $(n_1, n_2, n_3, \ldots, n_q)$ for different $\{n_i : 1 \leq i \leq q\}$ is still a problem . In this proposed paper, for d ∈ {0, 1, 2, 3}, we find a super (b, d)- labeling of edge-antimagic total of the star for subdivided $T(n, n, n + 4, n + 4, n5, \ldots, nq)$, where $q \geq 5$, $nr = 2r - 4(n + 3) + 1, 5 \leq r \leq q$ and $n \geq 3$ is odd. [12] detected the following bounds of lower and upper areas of the magic constant a for a specific subdivided stars subclass denoted by T (m, n, k):

Supporting theorem 2.1

If T (m, n, k) is a super (b, 0)- edge-antimagic total graph, then $\frac{1}{2l}(5l^2 + 3l + 6) \leq b \leq \frac{1}{2l}(5l^2 + 11l - 6)$, where $l = m + n + k$ are the bounds of lower and upper areas of the magic constant and are established by [8]

Supporting theorem 2.2

If $\underbrace{T\ (n, n, \ldots, n)}_{n\text{-times}}$ is a super (b, 0) - edge-antimagic total graph, then $\frac{1}{2l}$ $(5l^2 + (9-2n)\ l + n2 - n) \le b \le \frac{1}{2l}$ $(5l^2 + (2n + 5)\ l + n - n^2)$, where $l = n^2$.

For d = 0, [7] proved the bounds of lower and upper areas of the magic constant for the most extended subdivided stars subclasses denoted by $T\ (n1, n2, n3, \ldots, nq\ )$ with any $ni \geq 1$ for $1 \leq i \leq q$, which are presented in the following supporting theorem:

Supporting theorem 2.3

If T $(n_1, n_2, n_3, n_4, n_5, n_6, n_7, \ldots, n_q)$ is a super (b, 0)- edge-antimagic total graph, then $\frac{1}{2l}$ $(5l^2 + (9 - 2q\ )l + (q^2 - r\ )) \le b \le \frac{1}{2l}$ $(5l^2 + (5 + 2q\ )l - (q^2 - q))$, where $l = \sum_{i=1}^{q} ni$.For d ∈ {0, 1, 2, 3,4,5}, [13] derived the following bounds of lower and upper limits of a on the same subdivided stars class:

Supporting theorem 2.4

If T $(n_1, n_2, n_3, n_4, n_5, n_6, n_7, \ldots, n_q)$ has a super (b, d)- labeling of edge-antimagic total, then $\frac{1}{2l}$ $(5l^2 + q^2 - 2lq + 9l - q - (l-1)ld) \le a \le \frac{1}{2l}$ $(5l^2 - q^2 + 2lq + 5l + q - (l - 1)ld)$, where $l = \sum_{i=1}^{q} ni$ and d ∈ {0, 1, 2, 3,4,5}.

3. SUPER (B, D)-LABELING OF EDGE-ANTIMAGIC TOTAL FOR SUBDIVIDED STARS

In this section, for d different values, we prove the super (b, d) - labeling of edge-antimagic total existence on stars in subdivided class under some conditions.

Theorem 3.1.

For any odd n ≥ 3 and q ≥ 5, G ≅ $T\ (n, n, n + 4, n + 4, n5, \ldots, nq\ )$ admits a super (b, 0)- labeling of edge-antimagic total with $b = 2v + p - 1$ and a super (b, 2)-labeling of EAT with $b = v + p + 1$ where v = |V(G)|, $s = (2n + 8) + \sum_{m}^{q} = 5^{[2m-5(n+3)+1]}$ and $n\ r = 2r - 4(n + 3) + 1$ for 5 ≤ r ≤ q.

Proof. Let us denote the G vertices and edges as follows. The details are given below in equations (3.1), (3.2), (3.3), (3.4), (3.5), (3.6) and (3.7) ,

$$V(G) = \{f\} \cup \{x^{l_i}_{i} \mid 1 \le i \le q ; 1 \le l_i \le n_i\}, \qquad (3.1)$$

$$E(G) = \{f\ x^{1}_{i} \mid 1 \le i \le q\} \cup \{x^{l_i}_{i}\ x_i^{l_i+1} \mid 1 \le i \le r ; 1 \le l_i \le n_i - 1\}. \qquad (3.2)$$

If v = |V (G)| and e = |E (G)| then

$$v = (4n + 9) + \sum_{m=5}^{q}[2m - 4(n + 3) + 1] \qquad (3.3)$$

and

$$e = v - 1 \qquad (3.4)$$

Here, describe the labeling $: V(G) \to \{1, 2, \ldots, v\}$ as given below

$$\lambda(f) = (3n + 6) + \sum_{m=5}^{q}[2^{m-5}(n + 3) + 1] \quad (3.5)$$

For odd function $1 \le l_i \le n_i$, where i = 1, 2, 3, 4 and 5 ≤ i ≤ r, we define

$$\lambda(u) = \begin{cases} \frac{l_1+1}{2}, & \text{for } u = x^{l_1}_{1}, \\ (n+2) - \frac{l_2+1}{2}, & \text{for } u = x^{l_2}_{2}, \\ (n+1) + \frac{l_3+1}{2}, & \text{for } u = x^{l_3}_{3}, \\ (2n+7) - \frac{l_4+1}{2}, & \text{for } u = x^{l_4}_{4}. \end{cases} \qquad (3.6)$$

$\lambda(x^{li}_{i}) = (2n + 7) + \sum_{m=5}^{i}[2^{m-5}(n + 3) + 1] - \frac{li+1}{2}$ respectively.

For even functions, $1 \le l_i \le n_i$ and $\alpha = (2n + 6) + \sum_{m=5}^{i}[2^{m-5}(n + 3) + 1]$

And for i= 1,2,3,4 and 5≤i≤q, we describe

$$\lambda(u) = \begin{cases} \gamma + \frac{l_1}{2}, & \text{for } u = x^{l_1}_{1}, \\ (\gamma + n) - \frac{l_2}{2}, & \text{for } u = x^{l_2}_{2}, \\ (\gamma + n) + \frac{l_3}{2}, & \text{for } u = x^{l_3}_{3}, \\ (\gamma + 2n+4) - \frac{l_4}{2}, & \text{for } u = x^{l_4}_{4}, \end{cases} \qquad (3.7)$$

Hence

$\lambda\ (x^{li}_{i}) = (\gamma + 2n + 4) + \sum_{m=5}^{i}[2^{m-5}(n + 3)] - \frac{li}{2}$ respectively.

All edge-sums, set -sums created by the above formulas forms array of consecutive integers $\gamma+2$, $\gamma+3$, . . . , $\gamma+1+e$, where $s = \gamma + 2$. As a result, λ admits b (p, 1) - labeling of edge-antimagic vertex. Hence, by statement 1.1, λ can be stretched to a super (b, 0) - labeling of edge-antimagic total with $b = v + e + p = 2v + (2n + 7) + \sum_{m=5}^{q}[2^{m-5}(n + 3) + 1]$ and to a super (b, 2)

- labeling of edge-antimagic total with $b = v + 1 + p = v + (2n + 9) + \sum_{m=5}^{i}[2^{m-5}(n + 3) + 1]$.

Theorem 3.2.

For any odd function n ≥ 3 and q ≥ 5, G ≅ $T\ (n, n, n + 4, n + 4, n5, n6, n7, \ldots, nq\ )$ admits a super (b, 1)- labeling of edge-antimagic total with b = s + $\frac{3v}{2}$ if v is an even function, where $v = |V(G)|, p = (2n + 8) + \sum_{m=5}^{q}[2^{m-5}(n + 3) + 1]$ and $ns = 2s - 4(n + 3) + 1$ for 5 ≤ s ≤ q .

Proof. Describe V (G), E (G) and λ π: V (G) → {1, 2, 3, 4, . . . ,v} as in theorem 3.1. Thus, the set of edge-sums B = {b j ; 1 ≤ j ≤ e}, where $b\,j = (2n + 6) + \sum_{m=5}^{q}[2^{m-5}(n + 3) + 1] + j$ constitutes an array arithmetic with general difference 1. Consequently, the edge-labels set is C = {cj; 1 ≤ j ≤ e}, where cj = v + j. The set of edge-weights is defined as $D = \{b_{2i} - 1 + c_{e-i} + 1\ ;\ 1 \le i \le \frac{e+1}{2}\} \cup \{b_{2j} + c_{\frac{e-1}{2}-j+1}\ ;\ 1 \le j \le \frac{e+1}{2} - 1\}$. It is easy to see that D constitutes an array of arithmetic with d = 1. Since, all vertices obtain the labels of smallest, λ is a super (b, 1)- labeling of edge-antimagic total. In the following corollary, by supporting theorem 1.1 of duality, we can find another super (a, 0)- labeling of edge-antimagic total with dissimilar constant in magic as said in theorem 3.1.

Corollary 3.1.

For any odd n ≥ 3 and r ≥ 5, $G \cong T\ (n, n, n + 4, n + 4, n5, n6, n7, \ldots, nq)$ admits a super (a′, 0)-EAT labeling with $a' = 3v - (2n + 5) - \sum_{m=5}^{q}[2^{m-5}(n + 3) + 1]$, where $ns = 2s^{-4}(n + 3) + 1$ and 5 ≤ s ≤ q.

4. CONCLUSION

In this proposed paper, we have shown that a trees subclass, namely subdivided star denoted by T (n, n, n + 4, n + 4, n5, . . . , nr) admits a super (a, d)- labeling of edge-antimagic total for d = 0, 1, 2, when $n\,p = 2p - 4(n + 3) + 1$, r ≥ 5, 5 ≤ p ≤ r and n ≥ 3 is odd. However, for remaining n_i combinations, problem is still open.

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