DJ Journal of Engineering and Applied Mathematics, Vol. 1(1) 2015, pp. 4-16

RESEARCH ARTICLE

Controllability of Sobolev-Type Neutral Mixed Integrodifferential Evolution System in Banach Spaces

*B Radhakrishnan[1]

[1]Department of Mathematics, PSG College of Technology, Coimbatore-641004, Tamilnadu, India.

Received-13 October 2015, Revised-13 November 2015, Accepted-19 November 2015, Published-27 November 2015

ABSTRACT

In this paper, the author studied the controllability results for Sobolev type neutral mixed semilinear integrodifferential impulsive evolution systems with nonlocal initial conditions in Banach spaces. The results are obtained by using variations of constants formula and Schaefer's fixed point theorem.

Keywords: Controllability, Semilinear differential system, Integrodifferential system, Neutral impulsive system, Banach Spaces.

1. INTRODUCTION

The subject of differential equations has vast applications to the real world problems. Differential equations are used not only in physics but also in chemistry, biology, economics, engineering etc. The solutions of the differential equations are used to predict the behaviours of the system at a future time, or at an unknown location, or under some applied constraints. Integral equations form a very rich class of equations. The study of integrodifferential equations is relatively a new area in mathematics full of open problems that attracts an increasing level of interest. Differential and integrodifferential equations, especially nonlinear, present the most effective way for describing complex process. Most of the practical systems are integrodifferential equations in nature and hence the study of integrodifferential equations is very important. In general functional differential equations or evolution equations serve as an abstract formulation of many partial integrodifferential equations which arise in problems connected with heat-flow in materials with memory and many other physical phenomena.

It is well known that the systems described by partial differential equations can be expressed as abstract differential equations [1]. These equations occur in various fields of study and each system can be represented by different forms of differential or integrodifferential equations in Banach spaces. Using the method of semigroups, various solutions of nonlinear and semilinear evolution equations have been discussed by [1]. The study of abstract nonlocal semilinear initial value problems was initiated by [2, 3]. Since it is demonstrated that the nonlocal problems have better effects in applications than the classical Cauchy problems [4] established the existence of solutions of semilinear evolution equations of Sobolev type in Banach spaces. This type of equations arise in various applications such as in the flow of fluid through fissured rocks, thermodynamics, and shear in second-order fluids. For more details, we refer the reader to [5, 6, 7].

Controllability of linear and nonlinear systems represented by ordinary differential equations in finite-dimensional spaces has been extensively investigated. The problem of controllability of linear systems represented by differential equations in Banach spaces has been extensively studied by several authors [8]. Several papers have appeared on finite dimensional controllability of linear systems [9] and infinite dimensional systems in abstract

*Corresponding author. Tel.: +919976756881
Email address: radhakrishnanb1985@gmail.com (B.Radhakrishnan)
Double blind peer review under responsibility of DJ Publications
http://dx.doi.org/10.18831/djmaths.org/2015011002

spaces [10]. Of late the controllability of nonlinear systems in finite-dimensional spaces is studeid by means of fixed point principles [11]. Several authors have extended the concept of controllability to infinite-dimensional spaces by applying semigroup theory [12, 13].Controllability of nonlinear systems with different types of nonlinearity has been studied by many authors with the help of fixed point principles [14]. It is remarkable that [15,19,20] provide some sufficient conditions for controllability of integer functional evolution equations of Sobolev type by virtue of semigroup theory via the techniques of fixed point theorem.

From the above literatures, it should be noted that there are several contributions on the existence and controllability of Sobolev type equation using semigroup theory and existence and controllability of integrodifferential equations with and without randomness using one or more parameter families. Till now, there is no work reported on the exact controllability of Sobolev-type neutral impulsive mixed integrodifferential system using evolution operators. Motivated by this fact, in this paper, we make an attempt to fill this gap by studying controllability of Sobolev-type neutral mixed integrodifferential evolution system in Banach spaces.

2. PRELIMINARIES

Consider the class of sobolev-type semilinear neutral functional integrodifferential system with nonlocal conditions of the form

$$\frac{d}{dt}[Ex(t) - g(t, xt)] = A(t)x(t) + Bu(t) + f\left(t, x_t, \int_0^t k(t,s,x_s)ds, \int_0^b h(t,s,x_s)ds\right), t \in \mathcal{J} \quad (2.1)$$

$$x(t) + q(x) = \emptyset(t), t \in [-r, 0] \quad (2.2)$$

where the state variable x(.) takes values in a separable Banach space X with norm ‖.‖ and the control function u(.) is given in L^2(I,U), a Banach space of admissible control functions with U as a Banach space and the interval I = [0, b]. E and B is a bounded linear operator from U into X and A(t) : $Dt \subset X \to X$ generates an evolution system $\{U(t,s)\}0 \le s \le t \le b$ on the separable Banach space X. The functions $g : I \times C \to X, f : I \times C \times X \times X \to X, k : I \times I \times C \to X; h : I \times I \times C \to X, q : C(I,X) \to X$ are given functions. Here $C = C([-r, 0], X)$ is the Banach space of all continuous functions $\emptyset: [-r, 0] \to X$ endowed with the norm $\| \emptyset \| = \sup\{|\emptyset(\theta)| : -r \le \theta \le 0$. Also for $x \in C([-r; b], X)$ we have $xt \in C, for\ t \in [0, b], x_t(\theta) = x(t + \theta); for \theta \in [-r, 0]$. The norm of the X is denoted by ‖·‖.

Throughout this paper, $\{A(t) : t \in \mathbb{R}\}$ is a family of closed linear operators defined on a common domain D which is dense in X and we assume that the linear non-autonomous system

$$u'(t) = A(t)u(t), \quad s \le t \le b, \qquad u(s) = x \in X \quad (2.3)$$

has associated evolution family of operators $\{U(t,s) : 0 \le s \le t \le b\}$. In the next definition, L(X) is a space of bounded linear operators from X into X endowed with the uniform convergence topology.

Definition 2.1. A family of operators $\{U(t,s) : 0 \le s \le t \le b\} \subset L(X)$ is called a evolution family of operators for (3) if the following properties hold:

a. $U(t,s)U(s,\tau) = U(t,\tau)$ and $U(t,t)x = x$, for every $s \le \tau \le t$ and all $x \in X$;
b. For each $x \in X$; the function for $(t,s) \to U(t,s)x$ is continuous and $U(t,s) \in L(X)$ for every $t \ge s$ and
c. $For\ 0 \le s \le t \le b$, the function $t \to U(t,s),\ for\ (s,t] \in L(X)$; is differentiable with $\frac{\partial}{\partial t}U(t,s) = A(t)U(t,s)$.

Definition 2.2. [16, 17] System (1) - (2) is said to be controllable on the interval I, if for every initial functions $x_0 \in X$ and $x_1 \in X$, there exists a control $u \in L^2(I,U)$ such that the solution x(.) of (1) - (2) satisfies $x(0) = x_0$ and $x(b) = x_1$.

Definition 2.3. A solution $x(\cdot) \in C([0,b], X)$ is said to be a mild solution of $(2.1) - (2.2)$, then the following integral equation is satisfied.

$$x(t) + q(x) = \emptyset(t)\ t \in [-r, 0]$$

$$x(t) = E^{-1}U(t,0)[E\emptyset(0) - Eq(x) - g(0,x_0)] + E^{-1}g(t,x_t) + \int_0^t E^{-1}U(t,s)Bu(s)ds + \int_0^t E^{-1}U(t,s)Bu(s)ds + \int_0^t E^{-1}U(t,s)A(s)g(s,x_s)ds + \int_0^t E^{-1}U(t,s) f\left(s, x_s, \int_0^s k(s,\tau,x_\tau)d\tau\ , \int_0^b h(s,\tau,x_\tau)d\tau\right) ds, t \in I \quad (2.4)$$

We need the following fixed point theorem due to Schaefer [18].

Theorem: 2.4. Let E be a normed linear space. Let $F : E \to E$ be a completely continuous operator, that is, it is continuous and the image of any bounded set is contained in a compact set and let

$$\zeta(F) = \{x \in E : x = \lambda F x \; for\ some\ 0 < \lambda < 1\}.$$

Then either ζ(F) is unbounded or F has a fixed point.

To study the controllability problem, we assume the following hypotheses:

(H1) A(t) generates a strongly continuous semigroup of a family of evolution operators U(t; s) and there exist constants M1 > 0 such that

$$\| U(t,s) \| \leq M1, for\ 0 \leq s \leq t \leq b,$$

(H2) The exists a positive constant $0 < b_0 < b$ and, for each $0 < t \leq b_0$, there is a compact set $V_t \subset X$ such that

$$U(t,s)f\left(s, x_s, \int_o^s k|(s,\tau,x_\tau\right) d\tau, \int_0^b h(s,\tau,x_\tau)\, d\tau),$$

$$U(t,s\,)A(s)g(s,x_s), U(t,s)Bu(s) \in V_t and\ all\ 0 \leq \tau \leq s \leq b_0.$$

(H3) The Linear operator $W: L^2(I,U) \to X$ defined by

$$Wu = \int_0^b U\,(b,s)Cu(s)ds$$

has an inverse operator W^{-1} which takes values in $L^2(I,U)/kerW$ and exists a positive constant M_2 such that $\| BW^{-1} \| \leq M_2$.

(H4)(i) The function $g: I \times C \to X$ is continuous for a.e. $t \in I$ and there exists a positive constant $M_g > 0, L_g > 0$ such that

$$\| g(t,x_t) \| \leq M_g \| x_t \|,$$

and

$$\| g(0,x_0) \| \leq L_g$$

(ii) Also there exists a constant $M_A > 0$ such that,

$$\| A(t)g(t,x_1) \| \leq M_A \| x_t \|,$$

holds for $t \in I$

(H5) **(i)** For each $t \in I$, the function $k(t,s,.) : C \to X$ is continuous and, for each $x \in C$,the function $k(.,.,x): I \to X$ is strongly measurable.

(ii) There exists an integrable function $M_K: I \to [0,\infty)$such that,

$$\| k(t,s,x) \| \leq M_k(t,s)\Omega_1(\| x \|),$$

holds for $t \in I, x \in C$,where $\Omega_1: [0,\infty) \to [0,\infty)$.is a continuous nondecreasing function.

(H6) **(i)** For each $t \in I$, the function $h(t,s,.): C \to X$ is continuous and, for each $x \in C$ the function $h(.,.,x): I \to X$ is strongly measurable.

(ii) There exists an integrable function $M_h: I \to [0,\infty)$ such that,

$$\| h(t,s,x) \| \leq M_h(t,s)\Omega_2(\| x \|),$$

holds for $t \in I, x \in C$,where $\Omega_2: [0,\infty) \to [0,\infty)$ is a continuous nondecreasing function.

(H7) The function $f: I \times C \times X \times X \to X$ satisfies the Caratheodory conditions:

(i) For each $t \in I$,the function $f(t,.,.,.): C \times X \times X \times\!\to X$ is continuous and for each $(x,y,z) \in C \times X \times X$, the function $f(.,x,y,z): I \to X$ is strongly measurable.

(ii) There exists an integrable function $M_f: I \to [0,\infty)$ such that,

$$\| f(t,x,y,z) \| \leq M_f(t)\Omega_3(\| \mathrm{x} \| + \| \mathrm{y} \| + \| \mathrm{z} \|),$$

holds for $t \in I, x \in C$, and $y,z \in X$, where $\Omega_3: [0,\infty \to (0,\infty)]$. is a continuous nondecreasing function.

(H8) The function $q: PC(I,X) \to X$ is continuous and there exists a constant $M_q \geq 0$ such that

$$\| q(x) \| \leq M_q,$$

for $x \in X$

(H9) The following inequality holds:
The function

$$\widehat{m}(t) = max\begin{Bmatrix} 1, M_1 \| E^{-1} \| M_f(t), M_k(t,s), M_h(t,s), \\ \int_0^t \frac{\partial}{\partial t} M_k(t,s)ds, \int_0^b M_k(t,s)ds \end{Bmatrix}$$

satisfies

$$\int_0^b \widehat{m}(s)ds < \int_d^\infty \frac{ds}{s + 2\Omega_1(s) + 2\Omega_2(s) + \Omega_3(s)}$$

where

$$d = \| E^{-1} \| M_1[\| E\emptyset(0) \| + EM_q \| + L_g]$$

and

$$d_2 = M_2 \| E^{-1} \| \Big\{ \| x_1 \| + \| E^{-1} \| [M_g + M_1 L_g(t) + M_1 \| E^{-1} \| M_A bK + M_1 \| E^{-1} \| \int_0^b M_f(s)\Omega_3[K + \int_0^s M_k(s,\tau)\Big)\Omega(K)d\tau + \int_0^b M_k(s,\tau)\Omega_2(K)d\tau]ds\}$$

The analysis is carried out by means of equations from (2.1) to (2.4)

3. CONTROLLABILITY RESULT

Theorem:3.1. If

If the hypotheses $[H1] - [H9]$ are satisfied, them the system $(1) - (2)$ is controllable on I.

Proof. Using the hypotheses [H3] for an arbitrary function x(.), define the control

$$u(t) = W^{-1}[x_1 E^{-1}U(b,0)\big[E\phi(0) - E_q(x) - g(0,x_0)\big] - E^{-1}g(b,x_b) - \int_0^b E^{-1}U(b,s)A(s)g(s,x_{s)}\, ds - \int_0^b E^{-1}U(b,s)f(s,x_s, \int_0^s k(s,\tau,x_\tau)d\tau, \int_0^b h(s,\tau,x_\tau)d\tau)\, ds](t) \quad (3.1)$$

For $\phi \in C$, define $\widehat{\emptyset} \in C_b, C_b = C([-r,b],X)$ by

$$\widehat{\emptyset}(t) = \begin{cases} \emptyset(t) - q(x), \quad -r \leq t \leq 0, \\ E^{-1}U(t,0)[E\emptyset(0) - Eq(x)], 0 \leq t \leq b. \end{cases}$$

If $x(t) = y(t) + \widehat{\emptyset}(t), t \in [-r,b]$, it is easy to see that y satisfied

$$y0 = 0$$

$$E^{-1}g\big(t, y_t + \widehat{\emptyset}t\big) - E^{-1}U(t,0)g\left(0, \widehat{\emptyset}0 + \int_0^t E^{-1}U(t,s)A(s)g\big(s, y_s + \widehat{\emptyset}s\big)ds + \int_0^t E^{-1}U(t,\eta)CW^{-1}[x_1 - E^{-1}U(b,0)[E\phi(0) - Eq(x) - g\big(0,\hat{\phi}0\big)] - E^{-1}g\big(b, yb + \hat{\phi}b\big) - \int_0^b E^{-1}U(b,s)A(s)g\big(s, y_s + \hat{\phi}_s\big)ds - \int_0^b E^{-1}U(b,s)f(s, y_s + \hat{\phi}_s, \int_0^s k\big(s,\tau,y_\tau + \hat{\phi}_\tau\big)dt, \int_0^b h(s,\tau,y_\tau + \widehat{\emptyset_\tau})\, d\tau\right) ds,](\eta)d\eta + \int_0^t E^{-1}U(t,s)f(s, y_s + \widehat{\emptyset}_s, \int_0^s k\big(s,\tau,y_\tau + \widehat{\emptyset}_\tau\big)d\tau, \int_0^b h\big(s,\tau,y_\tau + \widehat{\emptyset}_\tau\big)d\tau)\, ds, t \in I.$$

if and only if x satisfies

$$x(t) = E^{-1}U(t,0)[E\emptyset(0) - Eq(x)] + E^{-1}g(t,x_t) - E^{-1}U(t,0)g(0,x_0) + \int_0^t E^{-1}U(t,s)A(s)g(s,x_s)ds + \int_0^t E^{-1}U(t,\eta)CW^{-1}[x_1 - E^{-1}U(b,0)[E\emptyset(0) - Eq(x) - g(0,x_0)] - E^{-1}g(b,x_b) - \int_0^b E^{-1}U(b,s)A(s)g(s,x_s)ds - \int_0^b E^{-1}U(b,s)f(s,x_s,\int_0^s k(s,\tau,x_\tau),$$

$$\int_0^b h\,(s,\tau,x_\tau)d\tau)ds,](\eta)d\eta + \int_0^t E^{-1}U(t,s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau,$$

$$\int_0^b h\,(s,\tau,x_\tau)d\tau)ds, t \in I.$$

and $x(t) = \phi(t) - q(x), t \in [-r,0]$.

Define $C_b^0 = \{y \in C_b : y_0 = 0\}$ and we now show that when using the control, the operator $F: C_b^0 \to C_b^0$, defined by

$$(Fy)(t) = 0, \qquad t \in [-r,0]$$

$$(Fy)(t) = E^{-1}g\big(t, y_t + \widehat{\emptyset}_t\big) - E^{-1}U(t,0)g\big(0,\widehat{\emptyset}_0\big) + \int_0^t E^{-1}U(t,s)\,A(s)g\big(s,y_s + \widehat{\emptyset}_s\big)ds + \int_0^t E^{-1}U(t,\eta)CW^{-1}[x_1 - E^{-1}U(b,0)[E\emptyset(0) - Eq(x) - g\big(0,\widehat{\emptyset}_0\big)] - E^{-1}g\big(b,y_b + \widehat{\emptyset}_b\big) - \int_0^b E^{-1}U(b,s)A(s)g(s,y_s + \widehat{\emptyset}_s)ds \int_0^b E^{-1}U(b,s)f(s,y_s + \widehat{\emptyset}_s, \int_0^s k\big(s,\tau,y_\tau + \widehat{\emptyset}_\tau\big)d\tau, \int_0^b h\big(s,\tau,y_\tau + \widehat{\emptyset}_\tau\big)d\tau)ds,]\,(\eta)d\eta + \int_0^t E^{-1}U(t,s)f(s,y_s + \hat{\phi}_s, \int_0^s k\big(s,\tau,y_\tau + \hat{\phi}_t\big)d\tau, \int_0^b h\big(s,\tau,y_\tau + \hat{\phi}_\tau\big)d\tau)ds, \qquad t \in I.$$

has a fixed point. This fixed point point is then a solution of equation $(3.1) - (3.2)$.

Clearly $x(b) = x_1$ which means that the control u steers the system $(1) - (2)$ from the initial function $\emptyset$ to x_1 in time b, provide we can obtain a fixed point of the nonlinear operator F.

In order to study the controllability problem of $(2.1) - (2.2)$, we introduce a parameter $\lambda \in (0,1)$ and consider the following system

$$\frac{d}{dt}\big[Ex(t) - g\big(t,x(t)\big)\big] = \lambda A(t)x(t) + \lambda Bu(t) + \lambda f\left(t, x(t), \int_0^t k\,\big(t,s,x(s)\big)ds, \int_0^b h\,\big(t,s,x(s)\big)ds\right), t \in [0,b], \tag{3.2}$$

$$x(t) + q(x) = \lambda\emptyset(t) \quad t \in [-r,0] \tag{3.3}$$

First we obtain a priori bounds for the mild solution of the equation (3.2)-(3.3). Then from

$$x(t) = \lambda E^{-1}U(t,0)[E\phi(0) - Eq(x)] + \lambda E^{-1}g(t,x_t) - \lambda E^{-1}U(t,0)g(0,x_0) + \lambda\int_0^t E^{-1}U(t,s)A(s)g(s,x_s)ds$$

$$+\lambda \int_0^t E^{-1}U(t,\eta)CW^{-1}[x_1 - E^{-1}U(b,0)[E\emptyset(0) - E_q(x) - g(0,x_0)]$$

$$-E^{-1}g(b,x_b) - \int_0^b E^{-1}U(b,s)A(s)g(s,x_s)ds$$

$$-\int_0^b E^{-1}U(b,s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau, \int_0^b h\,(s,\tau,x_\tau)d\tau)ds,](\eta)d\eta$$

$$+\lambda \int_0^t E^{-1}U(t,s)f(s,x_s, \int_0^s k(s,\tau,x_\tau)d\tau\,, \int_0^b h\,(s,\tau,x_\tau)d\tau)ds, t \in I.$$

we have

$$\| x(t) \| \leq \| E^{-1}U(t,0)[E\emptyset(0) - Eq(x)] \| + \| E^{-1}g(t,x_t) \| + \| E^{-1}U(t,0)g(0,x_0) \| + \int_o^t \| E^{-1}U(t,\eta)CW^{-1}[x_1 - E^{-1}U(b,0)[E\emptyset(0) - Eq(x) - g(0,x_0)] - E^{-1}g(b,x_b) - \int_0^b E^{-1}U(b,s)A(s)g(s,x_s)ds - \int_0^b E^{-1}U\,(b,s)f\left(s,x_s,\int_0^s k(s,\tau\, x_\tau\right)d\tau, \int_0^b h(s,\tau,x_\tau)d\tau)\,ds,](\eta) \| d\eta + \int_0^t \| E^{-1}U(t,s)f(s,x_s, \int_0^s k(s,\tau,x_\tau)d\tau, \int_0^b h(s,\tau,x_\tau)d\tau) \| ds \leq$$

$$\leq \| E^{-1} \| M_1[\| \emptyset(0) \| + M_q + \widehat{M}_q] + \| E^{-1} \| M_g K + M_1 \| E^{-1} \| \int_0^t M_A \| x_s \| ds + M_1 b d_2 + M_1 \| E^{-1} M_f(s)\Omega_3[\| x \| \int_0^t M_k(s,\tau)\Omega_1(\| x \|)d\tau + \int_0^b M_k(s,\tau)\Omega_2 (\| x \|)d\tau]ds$$

Let us take the right hand side of the above inequality as $\mu(t)$. Then we have $x(0) = \mu(0) = d$, and $\| x(t) \| \leq \mu(t)$, with

$$\mu'(t) = M_1 \| E^{-1}M_A \| x_t \| + M_1 \| E^{-1} \| M_f(s)\Omega_3[\| x \| + \int_0^t M_k(t,s)\Omega_1(\| x \|)ds + \int_0^b M_h(t,s)\,\Omega_2(\| x \|)ds] \leq d_1\mu(t) + M_1 \| E^{-1} \| M_f(s)\Omega_3[\mu(t) + \int_0^b M_k(t,s)\Omega_1\big(\mu(s)\big)ds + \int_0^b M_h(t,s),\Omega_2\big(\mu(s)\big)ds]$$

where $d_1 = M_1 \| E^{-1} \| M_A$. Since μ is obviously increasing, let

$$\omega(t) = \mu(t) + \int_0^t M_k(t,s)\Omega_1\big(\mu(s)\big)ds + \int_0^b M_h(t,s)\Omega_2\,\big(\mu(s)\big)ds$$

Then $\omega(0) = \mu(0) = c$ and $\mu(t) \leq \omega(t)$

$$\omega'(t) = \mu'(t) + M_k(t,s)\Omega_1\big(\mu(t)\big) + M_h(t,s)\Omega_2\big(\mu(t)\big) + \int_0^t \frac{\partial}{\partial s} M_k(t,s)\Omega_2\big(\mu(s)\big)ds$$

$$\leq \widehat{m}\{\omega(t) + \Omega_3\big(\omega(t)\big) + 2\Omega_1\big(\omega(t)\big) + 2\Omega_2\big(\omega(t)\big)\}$$

This implies

$$\int_{\omega(0)}^{\omega(t)} \frac{ds}{s + \Omega_3(s) + 2\Omega_1(s) + 2\Omega_2(s)} \leq \int_0^b \widehat{m}(s)ds \leq \int_d^\infty \frac{ds}{s + \Omega_3(s) + 2\Omega_1(s) + 2\Omega_{2(s)}}$$

This inequality shows that there is a priori bound $K > 0$ such that $\omega(t) \leq K$ and hence $\mu(t) \leq K, t \in [0,b]$. Since $\| x(t) \| \leq K, t \in I$,we have

$\| x \|_1 = \sup\{\| x(t) \| : -\gamma \le t \le b \| < K$

where K is depending only on b and the function

$M_f(.), M_k(.), M_h(.), \Omega_1(.), \Omega_2(.), \Omega_3(.)$.

Next we must prove that the operator F is a completely continuous operator. Let

$B_k = \{y \in C_b^0 : \| y \|_1 \le K\}$, for some $K \ge 1$.

We first show that the set $\{Fy : y \in B_k\}$ is equicontinuous. Let $y \in B_k$ and $t_1, t_2 \in [0, b]$.

Then if $0 < t_1 < t_2 \le b$, we have

$$\| (Fy)(t_1) - (Fy)(t_2) \|$$

$$\le \| E^{-1} \| \| g(t_1, y_{t1} + \widehat{\emptyset}_{t_1}) - g(t_2, y_{t_2} + \widehat{\emptyset}_{t_2}) \| + \| E^{-1} \| \| U(t_1, 0) - U(t_2, 0) \| \| g(0, \widehat{\emptyset}_0) \| + \int_0^{t_1} \| E^{-1} \| \| [U(t_1, \in) - U(t_2, \in)] U(\in, s) A(s) g(s, y_s + \widehat{\emptyset}_s) \| ds + (t_2 - t_1) M_1 \| E^{-1} \| M_A K' + \int_0^{t_1} \| E^{-1} \| \| [U(t_1, \in) - U(t_2, \in)] U(\in, \eta) C W^{-1} [x_1 -$$

$$E^{-1} U(b, 0)[E\phi(0) - Eq(x) - g(0, \widehat{\phi}_0)] - E^{-1} g(b, y_b + \widehat{\phi}_b) - \int_0^b E^{-1} U(b, s) A(s)\, g(s, y_s + \widehat{\phi}_s) ds - \int_0^b E^{-1} U(b, s) f(s, y_s + \widehat{\phi}_s, \int_0^s k(s, \tau, y_\tau + \widehat{\emptyset}_\tau) d\tau) ds,](\eta) \| d\eta + (t_2 - t_1) \| E^{-1} \| M_1 M_2 [x_1 + \| E^{-1} \| M_1 \widehat{M}_g + \| E^{-1} \| M_g K' + M_1 b \| E^{-1} \| M_A K' + M_1 b \| E^{-1} \| M_f(t)(K' + b M_k K' + b M_h K'),] + \| \int_0^{t_1} E^{-1} [U(t_1, s) - U(t_2, s)] f(s, y_s + \widehat{\emptyset}_s, \int_0^s k(s, \tau, y_\tau + \widehat{\emptyset}_\tau) d\tau, \int_0^b h(s, \tau, y_\tau + \widehat{\emptyset}_\tau) d\tau) ds \| + (t_2 - t_1) \| E^{-1} \| M_1 M_f(t)(K' + b M_k K' + b M_h K')$$

where $K' = K + \| \widehat{\emptyset} \|$. The right hand side is independent of $y \in B_K$ and tends to zero as $t_2 - t_1 \to 0$, since by the assumption (H2) implies the continuity in the uniform operator topology.

Thus the set $\{F_y, y \in B_K\}$ is equicontinuous.

Notice that we considered here only the case $0 < t_1 < t_2$, since the other cases $t_1 < t_2 < 0$ or $t_1 < 0 < t_2$ are very simple.

It is easy to see that the family FB_K is uniformly bounded. Next we show that $\overline{FB_K}$ is compact. Since we have shown that FB_K is an equicontinuous collection, it suffices, by the Arzela-Ascoli theorem, to show that F maps B_K into a precompact set in X.

Let $0 < t \le b$ be fixed and $\in$ a real number satisfying $0 < \in < t$. For $y \in B_K$, we define $(F_\in y)(t) = E^{-1} g(t, yt + \widehat{\emptyset}_t) - E^{-1} U(t, 0) g(0, \widehat{\emptyset}_0) + \int_0^{t\in} E^{-1} U(t, s) A(s) g(s, y_s + \widehat{\emptyset}_s) ds + \int_0^{t\in} E^{-1} U(t, \eta)\, C W^{-1} [x_1 - E^{-1} U(b, 0)[E\phi(0) - Eq(x) - g(0, \hat{\phi}_0)] - E^{-1} g(b, y_b + \hat{\phi}_b) - \int_0^b E^{-1} U(b, s) A(s) g(s, y_s + \hat{\phi}_s) ds -$

$\int_0^b E^{-1}U(b,s)f\,(s, y_s + \hat{\phi}_s, \int_0^s k\,(s,\tau, y_\tau + \hat{\phi}_\tau d\tau,$

$$\int_0^b h\left(s,\tau, y_\tau + \hat{\phi}_\tau\right)d\tau)ds,](\eta)d\eta + \int_0^{t_\epsilon} E^{-1}U(t,s)f(s, y_s + \hat{\phi}_s, \int_0^s k\left(s,\tau, y_\tau + \hat{\phi}_\tau\right)d\tau, \int_0^b h\,(s,\tau, y_\tau + \hat{\phi}_\tau)d\tau)ds.$$

Now, by the assumption $(H2)$, the set $Y_\epsilon(t) = \{(F_\epsilon y)(t): y\epsilon B_K\}$ is precompact in X for every ϵ, $0 < \epsilon < t$.Moreover, for every $y\epsilon B_K$ we have $\| (Fy)(t) - (F_\epsilon y)(t) \| \le \int_{t-\epsilon}^t \| E^{-1}U(t,s)A(s)g\left(s, y_s + \widehat{\emptyset}_s\right) \| ds + \int_{t-\epsilon}^t \| E^{-1}U(t,\eta)CW^{-1}[x_1 - E^{-1}U(b,0)\left[E\phi(0) - Eq(x) - g\left(0,\hat{\phi}_0\right)\right] - E^{-1}g\left(b, y_b + \hat{\phi}_b\right) - \int_0^b E^{-1}U\,(b,s)A(s)g\left(s, y_s + \hat{\phi}_s\right)ds - \int_0^b E^{-1}U\,(b,s)f(s, y_s + \hat{\phi}_s, \int_0^s k\left(s,\tau, y_\tau + \hat{\phi}_\tau, \int_o^b h\left(s,\tau, y_\tau + \hat{\phi}_\tau\right)d\tau\right)ds,](\eta) \| d\eta + \int_{t-\epsilon}^t \| E^{-1}U(t,s)f(s, y_s + \widehat{\emptyset}_s, \int_0^s k\left(s,\tau, y_\tau + \widehat{\emptyset}_\tau\right)d\tau, \int_0^b h(s,\tau, y_\tau + \widehat{\emptyset}_\tau)d\tau) \| ds.$

Therefore

$$\| (Fy)(t) - (F_\epsilon y)(t) \| \to 0$$

as $\epsilon \to 0$, and there are precompact sets arbitrarily close to the set $\{(Fy)(t): y\epsilon B_K\}$.

Hence the set $\{(Fy)(t): y\epsilon B_K\}$ is precompact in X.

It remains to be shown that $F: C_b^0 \to C_b^0$ is continuous. Let $\{y_n \|_0^\infty \subset C_b^0$ with $y_n \to y$ in C_b^0. Then there is an integer K such that $\| y_n(t) \| \le K$ for all n and $t \in I$, so $y_n \in B_K$ and $y \in B_K$. By (H4),(H7),$g\left(t, y_n(t) + \widehat{\emptyset}_t\right) \to g\left(t, y(t) + \widehat{\emptyset}_t\right)$, for each $t \in I$ and since $\| g\left(t, y_n(t) + \hat{\phi}_t\right) - g\left(t, y_n(t) + \hat{\phi}_t\right) \| \le 2M_gK'$,$A(t)g\left(t, y(t) + \widehat{\emptyset}_t\right)$, for each $t \in I$ and since$\| A(t)g\left(t, y_n(t) + \widehat{\emptyset}_t\right) - A(t)g(t, y(t) + \widehat{\emptyset}_t) \| \le 2M_AK'$,and $f\left(t, y_n(t) + \widehat{\emptyset}_t, \int_0^t k(t,s, y_n(s) + \widehat{\emptyset}_s\right)ds, \int_0^b h\,(t,s, y_n(s) + \widehat{\emptyset}_s)ds) \to f(t, y(t) + \widehat{\emptyset}_t, \int_0^t k\left(t,s, y(s) + \widehat{\emptyset}_s\right)ds, \int_0^b h(t,s, y(s) + \widehat{\emptyset}_s)ds),$

For each $t \in I$ and since $\| f(t, y_n(t) + \widehat{\emptyset}_t, \int_0^t k\left(t,s, y_n(s) + \widehat{\emptyset}_s\right)ds, \int_0^b h\left(t,s, y_n(s) + \widehat{\emptyset}_s\right)ds) - f(t, y(t) + \widehat{\emptyset}_t, \int_0^t k\left(t,s, y(s) + \widehat{\emptyset}_s\right)ds, \int_0^b h(t,s, y(s) + \widehat{\emptyset}_s)ds) \| \le 2\mu_K{}'(t), K' = K + \| \widehat{\emptyset} \|$, we have, by dominated convergence theorem,

$$\begin{aligned}\| (Fy_n)(t) - (Fy)(t) \| \le \| E^{-1} \| &\| \left[g\left(t, y_n(t) + \widehat{\emptyset}_t\right) - g\left(t, y(t) + \widehat{\emptyset}_t\right)\right] \\ &\| + M_1 \int_0^t \| E^{-1} \| [A(s)\,g\left(s, y_n(s) + \widehat{\emptyset}_s\right) - A(s)g(s, y(s) + \widehat{\emptyset}_s)]ds \\ &+ M_1M_2 \int_0^t \| E^{-1} \| [\| E^{-1} \| \| \left[g\left(b, y_n(b) + \widehat{\emptyset}_b\right) - g\left(b, y(b) + \widehat{\emptyset}_b\right)\right] \\ &\| + M_1 \int_0^b \| E^{-1} \| \| [A(s)\,g\left(s, y_n(s) + \widehat{\emptyset}_s\right) - A(s)g(s, y(s) + \widehat{\emptyset}_s)] \\ &\| ds \quad + M_1 \int_0^b \| E^{-1} \| \| [f(s, y_n(s) + \widehat{\emptyset}_s, \int_0^s k\left(s,\tau, y_n(\tau) + \widehat{\emptyset}_\tau\right)d\tau,\end{aligned}$$

$$\int_0^a h\left(s,\tau, y_n(\tau) + \widehat{\emptyset}_\tau\right)d\tau) - f(s, y(s) + \widehat{\emptyset}_s, \int_0^s k\left(s,\tau, y(t) + \widehat{\emptyset}_\tau\right)d\tau, \int_0^a h\left(s,\tau, y(\tau) + \widehat{\emptyset}_\tau\right)d\tau)] \| ds](\eta)d\eta$$

$$+M_1\int_0^t \| E^{-1} \| \| [f(s, y_n(s) + \hat{\phi}_s, \int_0^s k(s,\tau, y_n(\tau) + \hat{\phi}_\tau, \int_0^b h\left(s,\tau, y_n(\tau) + \hat{\phi}_\tau\right)d\tau) - f(s, y(s) + \hat{\phi}_s, \int_0^s k\left(s,\tau, y(\tau) + \hat{\phi}_\tau\right)d\tau)] \| ds$$

$$\to 0 \quad as \quad n \to \infty.$$

Thus F is continuous. This completes the proof that F is completely continuous.

Finally the set $\xi(F) = \{y \,\epsilon\, C_b^0 : y = \lambda F y, \lambda \,\epsilon\, (0,1)\}$ is bounded, since for every solution y in $\zeta(F)$, the function $x = y + \hat{\phi}$ is a mild solution of (3.2)-(3.3) for which we have proved that $\| x \|_1 \leq K$ and hence

$$\| y \|_1 \leq K + \| \hat{\emptyset} \|.$$

Consequently, by Schaefer's theorem, the operator F has a fixed point in C_0^b. This means that any fixed point of F is a mild solution of (2.1)-(2.2) and is controllable on I.

The analysis is carried out by means of equations in (3.1) to (3.3)

4. SOBOLEV TYPE NEUTRAL IMPULSIVE SYSTEMS

Consider the class of sobolev-type semilinear neutral functional impulsive integrodifferential system with nonlocal conditions of the form

$$\frac{d}{dt}\left[Ex(t) - g\left(t, x(t)\right)\right] = A(t)x(t) + Bu(t) + f(t, x(t), \int_0^t k\left(t, s, x(s)\right)ds, \int_0^b h\left(t, s, x(s)\right)ds) \quad t \in [0, b],$$

$$\Delta x(t_k) = I_k(x_{tk}), \quad k = 1,2, \dots, m,$$

$$x(t) + g(x) = \emptyset(t) \quad t \in [-r, 0] \qquad (4.1)$$

where the state variable $x(.)$ takes values in a separable Banach space X with norm $\|.\|$ and the control function $u(\,)$ is given in $\mathcal{L}^2(I, U)$, a Banach space of admissible control functions with U as a Banach space the interval $I = [0, b]$. E and B is a bounded linear operator from U into X and $A(t): D_t \subset X \to X$ generates an evolution system $\{U(t,s)\}_{0 \leq s \leq t \leq b}$ on the separable Banach space X. The functions $g: I \times C \to X$, $f: I \times C \times X \times X \to X$, $k: I \times I \times C \to X$ $h: I \times I \times C \to X$, $q: PC(I, X) \to X$ are given functions and $I_k: X \to X$ are appropriate functions and the symbol $\Delta x(t_k)$ represent the jump of the function u at t, which is defined by $\Delta x(t_k) = x(t^+) - x(t^-)$. Here $PC = PC([-r, 0], X)$ is the Banach space of all continuous functions $\emptyset : [-r, 0] \to X$ endowed with the norm $\| \emptyset \| = \sup\{|\emptyset(\theta)|: -r \leq \theta \leq 0\}$. Also for $x \in PC([-r, b], X)$ we have $x_t \in PC$ for $t \in [0, b], x_t(\theta) = x(t + \theta)$ for $\theta \epsilon [-r, 0]$. The solution of the (3.3) equation is given by

$$x(t) = E^{-1}U(t, 0)[E\emptyset(0) - Eq(x) - g(0, x_0)] + E^{-1}g(t, x_t) + \int_0^t E^{-1}U(t,s)Cu(s)ds + \int_0^t E^{-1}U(t,s)A(s)g(s, x_s)ds + \int_0^t E^{-1}U(t,s)f(s, x_s, \int_0^s k(s,\tau, x_\tau)d\tau, \int_0^b h(s,\tau, x_\tau)d\tau)ds + \sum_{0<t_i<t} B^{-1}S(t - t_k)I_k x(t_k), \quad t \in I.$$

$$\Delta x(t_k) = I_k\left(x_{t_k}\right), k = 1,2, \dots m,$$

$$x(t) + q(x) = \emptyset(t) \quad t \in [-r, 0]. \qquad (4.2)$$

In order to prove the main result we shall assume some additional hypothese:

(H10) The maps $I_k: X \to X$ are continuous and there exists a constant $I > 0$ such that

$\| I_k(x) \| \leq I \| x_t \|$, for $k \in \aleph$ and $x \in X$.

(H11) The following inequality holds:

The function

$$\widehat{m}(t) = \max\{1, M_1 \parallel E^{-1} \parallel M_f(t), M_k(t,t), \int_0^t \frac{\partial}{\partial t} M_k(t,s)ds, \int_0^b \frac{\partial}{\partial t} M_k\,(t,s)ds\}$$

satisfies

$$\int_0^b \widehat{m}(s)ds < \int_d^\infty \frac{ds}{s + 2\Omega_1(s) + 2\Omega_2(s) + \Omega_3(s)}$$

where

$$d = \parallel E^{-1} \parallel M_1[\parallel E\emptyset(0) \parallel + EM_q \parallel + L_g] + M_1 \sum_{k=1}^{m} I_k$$

and

$$d_2 = M_2 \parallel E^{-1} \parallel \Big\{\parallel x_1 \parallel + \parallel E^{-1} \parallel [M_g + M_1 L_g] + M_1 \parallel E^{-1} \parallel M_A bK + M_1 \parallel E^{-1} \parallel \int_0^b M_f(s)\Omega_3[K + \int_0^s M_k(s,\tau)\,\Omega(K)d\tau + \int_0^b M_h(s,\tau)\,\Omega_2(K)d\tau\Big] ds + M_1 \sum_{k=1}^{m} I_k\}.$$

Theorem: 4.1. If the hypotheses $[H1] - [H8], [H10] - [H11]$ are satisfied, then the system (4.1) is controllable on I.

Proof. Using the hypothese [H3] for an arbitrary function $x(.)$, define the control

$$u(t) = W^{-1}[x_1 - E^{-1}U(b,0)[E\phi(0) - Eq(x) - g(0,x_0)] - E^{-1}g(b,x_b) - \int_0^b E^{-1}U(b,s)A(s)g(s,x_s)ds - \int_0^b E^{-1}U(b,s)f(s,x_s, \int_0^s k(s,\tau,x_\tau)d\tau, \int_0^b h(s,\tau,x_\tau)d\tau)ds \qquad (4.3)$$

$$- \sum_{0<t_k<t} U\,(b,t_k)I_k(x_k)](t).$$

If $x(t) = y(t) + \hat{\phi}(t),\ t \in [-r,b]$, it is easy to see that y satisfied

$$y_0 = 0$$

$$y(t) = E^{-1}g(t, yt + \widehat{\emptyset}_t) - E^{-1}U(t,0)g(0,\widehat{\emptyset}_0) + \int_0^t E^{-1}U(t,s)A(s)g(s,y_s + \emptyset_s)ds + \int_0^t E^{-1}U(t,\eta)CW^{-1}[x_1 - E^{-1}U(b,0)[E\phi(0) - Eq(x) - g(0,\hat{\phi}_0)]$$

$$-E^{-1}g(b, yb + \hat{\phi}_b) - \int_0^b E^{-1}U(b,s)A(s)g(s,y_s + \hat{\phi}_s)ds \qquad (4.4)$$

$$-\int_0^b E^{-1}U(b,s)f(s,y_s + \hat{\phi}_s, \int_0^s k(s,\tau,y_\tau + \hat{\phi}_t)d\tau, \int_0^b h(s,\tau,y_\tau + \hat{\phi}_\tau + \hat{\phi}_\tau)d\tau)ds$$

$$-\sum_{0<t_k<b} U(b,t_k)I_K(yk+\hat{\phi}_k)]\,(\eta)d\eta +$$

$\sum_{0<t_k<t} U(t,t_k)I_k(yk+\hat{\phi}_k) + \int_0^t E^{-1}U(t,s)\,f(s,y_s+\hat{\phi}_s\,,\int_0^s k\big(s,\tau,y_\tau + \hat{\phi}_\tau\big)d\tau)\,ds,\ t\,\epsilon\,I$

if and only if x satisfies

$$x(t) =$$

$$E^{-1}U(t,0)[E\phi(0)-Eq(x)]$$

$$+E^{-1}g(t,x_t)-E^{-1}U(t,0)g(0,x_0)$$

$$+\int_0^t E^{-1}U(t,s)\,A(s)g(s,x_s)ds$$
$$+\int_0^t E^{-1}U(t,\eta)CW^{-1}[x_1$$
$$-E^{-1}U(b,0)[E\phi(0)-Eq(x)-g(0,x_0)]$$
$$-E^{-1}g(b,x_b)$$
$$-\int_0^b E^{-1}U(b,s)A(s)g(s,x_s)ds$$
$$-\int_0^b E^{-1}U(b,s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau,$$

$$\int_0^b h(s,\tau,x_\tau)d\tau)ds$$
$$-\sum_{0<t_k<b} U(t,t_k)I_k(x_k)](\eta)d\eta$$
$$+\sum_{0<t_k<t} U(t,t_k)I_k(x_k)$$
$$+\int_0^t E^{-1}U(t,s)f(s,x_s,\int_0^s k(s,\tau,x_\tau)d\tau,$$

$$\int_0^b h\,(s,\tau,x_\tau)d\tau)ds,\quad t\in I$$

and $x(t)=\emptyset(t)-q(x),\ \ t\in[-r,0]$.

Define $C_b^0=\{y\in C_b: y_0=0\}$ and we now show that when using the control, the operator $F^*: C_b^0 \to C_b^0$, defined by

$$(F^*y)(t)=0, \qquad t\in[-r,0]$$

$$(F^*y)(t) = E^{-1}g\big(t,yt+\widehat{\emptyset}_t\big)$$
$$-E^{-1}(t,0)g\big(0,\widehat{\emptyset}_0\big)$$
$$+\int_0^t E^{-1}\,U(t,s)A(s)g\big(s,y_s$$
$$+\widehat{\emptyset}_s\big)ds$$
$$+\int_0^t E^{-1}U(t,\eta)CW^{-1}[x_1$$
$$-E^{-1}U(b,0)\big[E\phi(0)-Eq(x)$$
$$-g\big(0,\hat{\phi}_0\big)\big]-E^{-1}g(b,y_b$$
$$+\hat{\phi}_b)-$$

$$\int_0^b E^{-1}U(b,s)A(s)g\big(s,y_s+\hat{\phi}_s\big)ds$$

$$-\int_0^b E^{-1}U(b,s)f(s,y_s$$
$$+\hat{\phi}_s,\int_0^s k(s,\tau,y_\tau$$
$$+\hat{\phi}_t)d\tau,\int_0^b h(s,\tau,y_\tau$$
$$+\hat{\phi}_\tau)d\tau)ds$$
$$-\sum_{0<t_k<b} U(t,t_k)I_k(yk$$
$$+\hat{\phi}_k)](\eta)d\eta$$
$$+\int_0^t E^{-1}U(t,s)f(s,y_s$$
$$+\hat{\phi}_s,\int_0^s k(s,\tau,y_\tau$$
$$+\hat{\phi}_\tau,\int_0^b h\,(s,\tau,y_\tau+\hat{\phi}_\tau)d\tau)ds$$
$$+\sum_{0<t_k<t} U(t,t_k)I_k\big(y_k$$
$$+\hat{\phi}_k\big),\ \ t\in I.$$

has a fixed point. This fixed point is, then a solution of (4.1). All the analysis is shown in equations from (4.1) to (4.4).

Clearly, $(F*y)(b)=x(b)=x_1$, which means that the control u steers the system (4.1) from the initial state x_0 to the final state x_0 to the final state x_1 at time b, provided we can obtain a fixed point of nonlinear operator F^*. The remaining part of the proof is similar to theorem 3.1 and hence, it is omitted.

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